The Mortgage Credit Channel of Macroeconomic Transmission*

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November 23, 2016

Abstract

I investigate how the structure of the mortgage market influences macroeconomic dynamics, using a general equilibrium framework with prepayable debt and a limit on the ratio of mortgage payments to income. This realistic environment amplifies transmission from interest rates into debt, house prices, and economic activity. Monetary policy can more easily stabilize inflation due to this amplification, but contributes to larger fluctuations in credit growth. A relaxation of payment-to-income standards appears essential to the recent boom. A cap on payment-to-income ratios, not loan-to-value ratios, is the more effective macro-prudential policy for limiting boom-bust cycles.

1 Introduction

Mortgage debt is central to the workings of the modern macroeconomy. The sharp rise in residential mortgage debt at the start of the twenty-first century in the US and countries around the world has been credited with fueling a dramatic boom in house prices and consumer spending. At the same time, high levels of mortgage debt and household leverage have been blamed for the severity of the subsequent bust. Since mortgage credit evolves endogenously in response to economic conditions, its critical position in the macroeconomy raises a number of important questions. How, if at all,

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*This paper is a revised version of Chapter 1 of my Ph.D. dissertation at NYU. I am extremely grateful to my thesis advisors Sydney Ludvigson, Stijn Van Nieuwerburgh, and Gianluca Violante for their invaluable guidance and support. The paper benefited greatly from conversations with Andreas Fuster, Mark Gertler, Andy Haughwout, Malin Hu, Virgiliu Midrigan, Jonathan Parker, Johannes Stroebel, among many others, conference discussions by Amir Sufi, Paul Willen, and Hongjun Yan, and numerous insightful comments from seminar audiences. I thank eMBS for their generous provision of data, and NYU and the Becker-Friedman Institute for financial support.

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does mortgage credit growth propagate and amplify macroeconomic fluctuations in general equilibrium? How does mortgage finance affect the ability of monetary policy to influence economic activity? Finally, what role did changing credit standards play in the boom, and how might regulation have limited the resulting bust?

These questions all center on what I will call the mortgage credit channel of macroeconomic transmission: the path from primitive shocks, through mortgage credit issuance, to the rest of the economy. Characterizing this channel is challenging due to the complex links between mortgage debt and the macroeconomy. Large numbers of heterogeneous households participate in mortgage markets, both as borrowers and savers, trading history-dependent streams of cash flows that differ widely in interest rates. Mortgage contracts are specified in nominal terms, so that real mortgage payments are influenced by inflation. Taking out new mortgage debt is a costly process that typically requires prepayment of existing debt. Household decisions about whether and when to prepay existing mortgages respond endogenously to economic conditions as interest rates and house prices change. New borrowing is constrained by multiple limits determined by endogenous variables such as house prices and borrower incomes.

In this paper I develop a tractable modeling framework that embeds these features in a New Keynesian dynamic stochastic general equilibrium (DSGE) environment. The framework centers on two key mechanisms that define the mortgage credit channel.

First, at the intensive margin, new borrowing is limited by two factors: the ratio of the size of the loan to the value of the underlying collateral (“loan-to-value” or “LTV”), and the ratio of the mortgage payment to the borrower’s income (“payment-to-income” or “PTI”). While a vast literature documents the impact of LTV constraints on debt dynamics, the influence of PTI limits on the macroeconomy remains relatively unstudied, despite their central role in underwriting in the US and abroad. As I will show, PTI limits fundamentally alter the dynamics of mortgage credit growth, played an essential part in the boom and bust, and are likely to increase further in importance as an important feature of new mortgage regulation. Since in a heterogeneous population an endogenous and time-varying fraction of individuals will be limited by each constraint, I develop an aggregation procedure to capture these dynamics at the macro level and calibrate them to match loan-level microdata.

The payment-to-income ratio is also commonly known as the “debt-to-income” or “DTI” ratio. I use the term “payment-to-income” for clarity, since under either name the ratio measures the flow of payments relative to a borrower’s income, not the stock of debt relative to a borrower’s income.
Second, at the extensive margin, borrowers choose whether to prepay their existing loans and replace them with new loans, a process that incurs a transaction cost. This mechanism is designed to capture two empirical facts: only a small minority of borrowers obtain new loans in a given quarter, but the fraction that choose to do so is volatile and highly responsive to interest rate incentives. These dynamics stand in sharp contrast to traditional macro-housing models, in which debt levels mechanically track credit limits, and do not respond independently to interest rate incentives. I develop a method to tractably aggregate over the discrete prepayment decision, which I calibrate to match estimates from a workhorse prepayment model, and show that the endogenous response of prepayment to interest rates is of first-order importance for credit dynamics and transmission.

This framework generates two main sets of findings. The first set relate to interest rate transmission, where I find that the novel features of the model, when calibrated to US mortgage microdata, greatly amplify the influence of nominal interest rates on debt, house prices, and economic activity. The initial step in the transmission chain is that PTI limits are themselves highly sensitive to nominal interest rates, with an elasticity near 8. But because only a minority of borrowers are constrained by PTI at equilibrium, this would not by itself generate large aggregate effects. Instead, the key is the constraint switching effect, a novel propagation mechanism through which changes in which of the two constraints is binding for borrowers translate into large movements in house prices. This effect is quantitatively powerful, causing price-rent ratios to rise by more than 4% in response to a 1% fall in nominal rates. Rising house prices in turn loosen borrowing constraints for the LTV-constrained majority of the population, leading to more than twice the increase of credit growth relative to an alternative economy with an LTV constraint alone.

For transmission into output, borrowers’ option to prepay their loans turns out to be critical, due to its influence on the timing of credit growth. When borrowers can choose to prepay, a fall in rates leads to a wave of prepayments, new issuance, and new spending on impact, generating a large output response — a phenomenon I call the frontloading effect. Quantitatively, this effect amplifies the impact of a 1% technology shock on output by nearly 60%. Alternative economies without endogenous prepayment generate much slower issuance of credit with virtually zero effect on output.

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2 See Figure A.4 in the appendix.
3 Traditional models use one-period debt and assume that borrowers are always at their constraints, so that debt is equal to the debt limit at all times. Improvements to add persistence to debt limits or account for ratchet effects, as in Justiniano, Primiceri, and Tambalotti (2015), are more realistic but still imply that debt is a mechanical function of past debt limits.
despite a similar increase in debt limits. These results on transmission have important consequences for monetary policy, which is more effective at stabilizing inflation due to these forces, but contributes to larger swings in credit growth, posing a potential trade-off for central bankers concerned with stabilizing both markets.

The second set of findings relate to credit standards and the sources of the recent boom and bust, where I find that a relaxation of PTI limits was essential. While existing models that ignore the PTI constraint are able to produce large booms by loosening LTV limits, I find that a relaxation of LTV standards alone could not have created the observed boom if PTI limits had been held fixed at their historical standards. In contrast, an experiment calibrated to empirical evidence showing massive relaxation of PTI standards generates a realistic boom accounting for nearly half of the observed increase in price-rent and debt-household income ratios.

While a liberalization of PTI constraints is partially sufficient for explaining the boom, it also appears necessary for other factors to have played as large a role as they did. A simultaneous relaxation of PTI more than doubles the contribution of LTV liberalization to debt-household income growth, and causes LTV liberalization to increase, rather than decrease, price-rent ratios. For an alternative benchmark, an expected increase to housing utility that can generate virtually the entire boom when PTI limits are absent is severely dampened when PTI limits are present, cutting the rise in debt-household income ratios by a factor of four. These results have important implications for macroprudential regulation, implying that a cap on PTI ratios, not LTV ratios, is the more effective policy for limiting boom-bust cycles.

This paper builds on several existing strands of the literature. On the empirical side, it relates to a large and growing body of work demonstrating important links among mortgage credit, house prices, and economic activity, and documenting patterns of credit growth in the boom. Particularly relevant is Boldin (1993), who finds econometric evidence that changes in mortgage affordability due to movements in interest rates have strong effects on housing demand. My study complements these works by analyzing the theoretical mechanisms behind these links in general equilibrium.

Turning to theoretical models, the literature can be broadly split into two camps. The first comprises heterogeneous agent models, which often include rich specifica-
tions of idiosyncratic risk, costly financial transactions, and long-term mortgage contracts, but cannot tractably incorporate inflation, monetary policy, and endogenous output in general equilibrium. In contrast, a set of monetary DSGE models with housing and collateralized debt can easily handle these macroeconomic features, but use simplified loan structures that rule out important features of debt dynamics. In this paper I seek to combine these two approaches, embedding a realistic mortgage structure in a tractable general equilibrium environment.

Moreover, to my knowledge, Corbae and Quintin (2013) offer the only other macroeconomic model that incorporates a PTI constraint and uses its relaxation as a proxy for the housing boom. However, these authors use the PTI constraint to explore the relationship between endogenously priced default risk and credit growth in a model with exogenous house prices. While their setup delivers important findings regarding default and foreclosure, both absent from my model, these authors do not study the implications of the PTI constraint for interest rate transmission, or, through its influence on house prices, on the LTV constraint — the key to the results of this paper.

This work is also related to research connecting a relaxation of credit standards to the recent boom-bust. My findings largely support the importance of credit liberalization in the boom, with the specific twist that a relaxation of PTI constraints appears key. Of particular relevance is Justiniano et al. (2015), who find that the interaction of an LTV constraint with an exogenous lending limit can generate strong effects of movements in the non-LTV constraint on debt and house prices — a result echoed in many of the findings of this paper. By utilizing an endogenous PTI constraint in place of an exogenous fixed limit on lending, I am able to connect these dynamics to interest rate transmission, calibrate to observed relaxations of PTI standards in the data, and analyze the effects of the regulatory cap on PTI limits imposed by Dodd-Frank.

Finally, this paper parallels research on the redistribution channel of monetary policy. When borrowers hold adjustable-rate mortgages, changes in interest rates lead to changes in payments on the existing stock of debt, influencing borrower spending. This channel is separate from, and complementary to, the mortgage credit channel,
which operates instead through the flow of new credit driven by changes in borrowing constraints. Interestingly, while allowing borrowers to prepay their loans does allow for substantial changes in payments when interest rates fall, and therefore large re-distributions between borrowers and savers, the redistribution channel is nonetheless weak in my framework, leading to very small aggregate stimulus. The key difference is in the timing: under fixed-rate mortgages, changes in interest payments occur too slowly to influence output.

The remainder of the paper is organized as follows. Section 2 provides a simple example and presents facts from the data. Section 3 constructs the theoretical model, while Section 4 describes the calibration. Section 5 presents the results on interest rate transmission, and the consequences for monetary policy. Section 6 discusses the role of credit standards in the boom-bust, and the implications for macroprudential policy. Section 7 concludes. Additional results and extensions can be found in the appendix.

2 Background: LTV and PTI Constraints

This section presents a simple numerical example, and demonstrates the empirical properties of LTV and PTI limits in the data.

2.1 Simple Numerical Example

To provide intuition for model’s core mechanisms, I present a simplified example from an individual borrower’s perspective. Consider a prospective home-buyer who prefers to pay as little as possible in cash today, perhaps because she must save for the down payment and delaying purchase is costly. This borrower’s annual income is $50k, and she faces a 28% PTI limit, meaning that she can put at most $1.2k per month toward her mortgage payment. At an interest rate of 6%, this maximum payment is associated with a loan size of $160k, which is the most she can borrow subject to her PTI limit. Her maximum LTV ratio is 80% so that, including the minimum 20% down payment, she reaches her maximum loan size at at a house price of $200k.

This $200k house price represents the threshold at which the borrower switches from being LTV-constrained to PTI-constrained. This creates a kink in the borrower’s required down payment as a function of house price, shown as the solid blue line in Figure 1. Below this threshold price, the borrower is constrained by the value of her PTI limit, and above this threshold, she is constrained by her LTV limit.

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10 For this example, I abstract from property taxes, insurance, and non-mortgage debt payments, and round quantities to the nearest $1k = $1,000.
collateral. In this region, increasing her house value by $1 allows her to borrow an additional 80 cents, requiring her to pay only 20 cents more in down payment. But above the kink, she is constrained by her income, cannot obtain any additional debt no matter how valuable her collateral, and must pay for any additional housing in cash. This discrete change around the kink implies that a “corner solution” price of exactly $200k is a likely optimum for this borrower, and for the sake of the example, let us assume that this is indeed her choice.

From this starting point, imagine that the mortgage interest rate now falls from 6% to 5%, displayed as the dashed lines in Figure 1a. While the borrower’s maximum monthly payment has not changed, at a lower interest rate this $1.2k payment is now associated with a larger loan of $178k. But because of her LTV constraint, the borrower can only take advantage of this larger loan limit if she obtains a more valuable house as collateral. This shifts the kink in the down payment function to the right, with the threshold price now occurring at $223k — an 11% increase. If the borrower once again chooses her threshold house size, the result is a substantial increase in demand, potentially contributing to a large rise in house prices if others do the same. Note that this result depends crucially on the interaction of the LTV and PTI constraints, and would not be present under either constraint in isolation.

This example can also be used to analyze changes in credit standards. First, consider an increase in allowed PTI ratios. Since this intervention increases the maximum PTI loan size, the impact on the down payment function is the same as if the interest rate had fallen. Specifically, a rise from a 28% to a 31% PTI ratio exactly replicates the change in Figure 1a, once again raising the threshold house price, and potentially
boosting housing demand. In contrast, an increase in the maximum LTV ratio from 80% to 90%, shown in Figure 1b, has a starkly different impact. In this case, the borrower’s maximum loan size given her income is unchanged, at $160k. But with only a 10% down payment, the house price associated with this loan falls to $178k, an 11% decrease. If the borrower once again follows her corner solution, the result is a decrease in her housing demand, potentially contributing to a decline in house prices.

To understand this result, note that prior to the LTV loosening, moving from a $200k house to a $178k house would have let the borrower keep only $4.4k in cash, since she would have been forced to cut her loan size. But after the relaxation, the borrower can keep the entire $22k difference in cash, making the less expensive house much more tempting. Alternatively, note that a relaxation of the LTV limit increases the supply of collateral, since each unit of housing can collateralize more debt, but not the demand for collateral, since the borrower’s overall loan size cannot increase, leading to a fall in the price of collateral. This result reverses the implications of models in which borrowers face only an LTV constraint, where lower down payments typically increase housing demand and house prices.

### 2.2 LTV and PTI in the Data

This section considers the empirical properties of the LTV and PTI constraints. Figure 2 shows the distribution of combined LTV (CLTV) and PTI on newly issued conventional fixed-rate mortgages securitized by Fannie Mae for two points in time: the height of the boom (2006 Q1) and a recent post-crash date (2014 Q3).\(^{11}\) The CLTV plots display two patterns of interest. First, the influence of LTV limits is obvious, with the majority of borrowers grouped in large spikes at known institutional limits.\(^{12}\) Second, the cross-sectional distribution of CLTV changes little between 2006 and 2014, showing no major change in credit standards between the boom and post-crash environment.

Turning to the PTI plots, we observe markedly different patterns. While the distributions do not display large individual spikes as in the CLTV case, the clear influence of the institutional limit (45%) can be seen in the 2014 data, as the distributions build toward this limit before undergoing nearly complete truncation. The appearance of this smooth shape, rather than a single spike, likely stems from search frictions. Many

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\(^{11}\)Combined LTV is the ratio of total mortgage debt to the value of the house, summing if necessary over multiple mortgages against the same property. Identical plots using Freddie Mac data can be seen in Figure A.1 in the appendix.

\(^{12}\)The largest spikes occur at 80%, where borrowers must start paying for private mortgage insurance.
Figure 2: Fannie Mae Data: CLTV and PTI on Newly Originated Mortgages

Note: Histograms are weighted by loan balance. Source: Fannie Mae Single Family Dataset.
borrowers may prefer the threshold price described in Section 2.1, but are unable to find a house at precisely this value. If borrowers are willing to buy a house below but not above the threshold price, the joint pattern of LTV spikes and a truncated PTI distribution will emerge naturally. The distribution of cash-out refines — where borrowers remain in their existing homes and do not search — bolsters this argument, displaying much more PTI concentration near the institutional limit, but less bunching in CLTV. As a result, the 2014 data indicate that a nontrivial minority of borrowers are influenced by PTI limits.

In sharp contrast, the 2006 data display no evidence of a PTI limit at any level. Instead, the PTI histogram displays a smooth shape until 65% of pre-tax borrower income is committed to recurring debt payments, at which point the data are top-coded by the provider. While pre-boom data is not yet publicly available, this pattern is consistent with a massive loosening of PTI limits, which should have been much tighter before the boom than in 2014, due to a lower standard limit (36% instead of 45%) and higher interest rates. Comparison with the evolution of CLTV distributions implies that PTI standards likely experienced the more dramatic liberalization during the boom.

3 Model

This section constructs the model and shows its key equilibrium conditions.

3.1 Demographics and Preferences

The economy consists of two families, each populated by a continuum of infinitely-lived households. The households in each family differ in their preferences: one family contains relatively impatient households named “borrowers,” denoted with subscript $b$, while the other family contains relatively patient households named “savers,” de-

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13Bank preapproval letters often cap the price at which a buyer can make an offer to exactly this threshold price by default, potentially explaining this asymmetry.
14The public data goes back only to 2000, at which point loose enforcement of PTI limits is already observed. The liberalization of PTI limits likely occurred over the mid-1990s due to changes in federal GSE policy, such as the “GSE Act” of 1992, while the boom in price-rent ratios begins in 1997 Q3.
15Further evidence for this shift in PTI standards can be found in Figure A.5 of the appendix, which shows the evolution of quantiles of the PTI ratios on purchase loans for the period 2000-2014. Using Fannie Mae data, Pinto (2011) calculates that the 75th percentile of the PTI distribution over the period 1988-1991 was below 36%. Figure A.5d shows that by 2000, the 75th percentile has already reached 42%, and eventually peaks at 49%. In contrast, CLTV ratios are flat or falling over the boom, again suggesting a smaller change in LTV standards relative to PTI standards.
noted with subscript \( s \). The measures of the two populations are \( \chi_b \) and \( \chi_s = 1 - \chi_b \), respectively. Households trade a complete set of contracts for consumption and housing services within their own family, providing perfect insurance against idiosyncratic risk, but cannot trade these securities with members of the other family. Both types supply perfectly substitutable labor.

Each agent of type \( j \in \{ b, s \} \) maximizes expected lifetime utility over nondurable consumption \( c_{j,t} \), housing services \( h_{j,t} \), and labor supply \( n_{j,t} \)

\[
E_t \sum_{k=0}^{\infty} \beta_j^k u(c_{j,t+k}, h_{j,t+k}, n_{j,t+k})
\]  

(1)

where utility takes the separable form

\[
u(c, n, h) = \log(c) + \xi \log(h) - \eta \frac{n^{1+\varphi}}{1+\varphi}.
\]  

(2)

Preference parameters are identical across types with the exception that \( \beta_b < \beta_s \), so that borrowers are less patient than savers. For notation, define the marginal utility and stochastic discount factor for each type by

\[
u^c_{j,t} = \frac{\partial u(c_{j,t}, n_{j,t}, h_{j,t})}{\partial c_{j,t}} \quad \Lambda_{j,t+1} = \beta_j \frac{\nu^c_{j,t+1}}{\nu^c_{j,t}}
\]

with symmetric expressions for \( u^n_{j,t} \) and \( u^h_{j,t} \).

### 3.2 Asset Technology

For notation, stars (e.g., \( q^*_{t} \)) differentiate values for newly originated loans from the corresponding values for existing loans in the economy — a distinction necessary under long-term fixed-rate debt. The symbol “$” before a quantity indicates that it is measured in nominal terms.

The essential financial asset in the paper, and the only source of borrowing in the model economy, is the mortgage contract, whose balances (long for the saver, short for the borrower) are denoted \( m \). The mortgage is a nominal perpetuity with geometrically declining payments, as in Chatterjee and Eyigungor (2015). I consider a fixed-rate mortgage contract, which is the predominant contract in the US, but extend the model for the case of adjustable-rate mortgages in the appendix. Under the fixed-rate mortgage contract, the saver gives the borrower $1 at origination. In exchange,
the saver receives $(1 - \nu)^k q_t^*$ at time $t + k$, for all $k > 0$ until prepayment, where $q_t^*$ is the equilibrium coupon rate at origination, and $\nu$ is the fraction of principal paid each period.

As is standard in the US, mortgage debt is prepayable, meaning that the borrower can choose to repay the principal balance on a loan at any time, which cancels all future payments of the loan. If a borrower chooses to prepay her loan, she may choose a new loan size $m_{i,t}^*$ subject to her credit limits (defined below). Obtaining a new loan incurs a transaction cost $\kappa_{i,t} m_{i,t}^*$, where $\kappa_{i,t}$ is drawn i.i.d. across individual members of the family and across time from a distribution with c.d.f. $\Gamma_k$. This heterogeneity is needed to match the data, as otherwise identical model borrowers must make different prepayment decisions so that only an endogenous fraction prepay in each period. The borrower’s optimal policy is to prepay the loan if and only if her cost $\kappa_{i,t}$ is below some threshold value $\bar{\kappa}_t$, which therefore completely characterizes prepayment policy.

To allow for aggregation, I make a simplifying assumption: as part of the mortgage contract, borrowers must precommit to a prepayment rule for $\bar{\kappa}_t$ that depends only on aggregate states and the cost draw $\kappa_{i,t}$, and not on the characteristics of their individual loans. This implies that the unconditional probability of prepayment (prior to the draws of $\kappa_{i,t}$) is constant across borrowers at any single point in time. While this structure abstracts from cross-sectional dynamics, the prepayment rate will still endogenously respond to key macroeconomic conditions such as the average difference in rates between existing and new loans, the amount of home equity available to be extracted, and forward looking expectations of all aggregate variables.\footnote{Since I calibrate to match the average prepayment rate and prepayment sensitivity to interest rates, I should be able to eliminate bias in prepayment rates due to this assumption on average. As a result, bias should only arise from ignoring time variation in the shape of the distribution of interest rates and maturities.}

Turning to credit limits, a new loan for borrower $i$ must satisfy both an LTV and a PTI constraint, defined by

$$\frac{m_{i,t}^*}{p_h^* h_{i,t}^*} \leq \theta^{ltv} \quad \frac{(q_t^* + \alpha) m_{i,t}^*}{\omega i_{i,t} e_i t + \omega} \leq \theta^{pti}$$

where $m_{i,t}^*$ is the balance on the new loan, and $\theta^{ltv}$ and $\theta^{pti}$ are the maximum LTV and PTI ratios, respectively. These constraints are treated as institutional, and are not the outcome of any formal lender optimization problem.\footnote{This choice is motivated by the observation that industry standards for these ratios have persisted for decades, despite large changes in economic conditions.} The LTV ratio divides the loan balance by the borrower’s house value, given by the product of house price $p_h^*$ and the

$$\frac{m_{i,t}^*}{p_h^* h_{i,t}^*} \leq \theta^{ltv} \quad \frac{(q_t^* + \alpha) m_{i,t}^*}{\omega i_{i,t} e_i t + \omega} \leq \theta^{pti}$$
quantity of housing purchased $h_{i,t}^*$. For the PTI ratio, the numerator is the borrower’s initial payment, where $\alpha$ is an adjustment for property taxes, insurance, and servicing costs, while the denominator is the borrower’s labor income, equal to the product of the wage $w_t$, labor supply $n_{i,t}$, and an idiosyncratic labor efficiency shock $e_{i,t}$, drawn i.i.d. across borrowers and time with mean equal to unity and c.d.f. $\Gamma$. This income shock serves to generate variation among borrowers, so that an endogenous fraction is limited by each constraint at equilibrium.\(^{18}\) Finally, the offsetting term $\omega$ adjusts for the convention that the numerator of PTI typically includes payments on all recurring debt, including car loans, student loans, etc., by assuming that these payments require a fixed fraction of borrower income.\(^{19}\)

These expressions imply the maximum debt balances

$$m_{ltv,i,t}^* = \theta_{ltv} p_t h_{i,t}^*$$

$$m_{pti,i,t}^* = \frac{(\theta_{pti} - \omega) w_t n_{i,t} e_{i,t}}{q_t^* + \alpha}$$

consistent with each of the two constraints. Since the borrower must satisfy both constraints, her overall debt limit is $m_t^* \leq m_{i,t} = \min(m_{ltv,i,t}^*, m_{pti,i,t}^*)$. This constraint is applied at origination of the loan only, so that borrowers are not forced to delever if they violate these constraints later on. At equilibrium, this constraint will bind for newly issued loans, consistent with Figure 2, which shows few unconstrained borrowers at origination. However, households typically go years between prepayments in the model, during which time they are typically away from their borrowing constraints and accumulating home equity.

In addition to mortgages, households can trade a one-period nominal bond, whose balances are denoted $b_t$. One unit of this bond costs $1$ at time $t$ and pays $R_t$ with certainty at time $t+1$. This bond is in zero net supply, and is used by the monetary authority as a policy instrument. Since the focus of the paper is on mortgage debt, I assume that positions in the one-period bond must be non-negative, so that it is traded by savers only at equilibrium.

The final asset in the economy is housing, which produces a service flow each period equal to its stock, and can be owned by both types. A constant fraction $\delta$ of house value must be paid as a maintenance cost at the start of each period. Borrower and saver stocks of housing are denoted $h_{b,t}$ and $h_{s,t}$, respectively. To simplify the

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\(^{18}\)While I model $e_{i,t}$ as an income shock, it could stand in for any shock that varies the house price to income ratio in the population. Without variation in this ratio, all borrowers would be limited by the same constraint in a given period.

\(^{19}\)Since the dynamics of other debt are beyond the scope of this paper, I assume this debt is owed to other borrowers, so that it has no other influence beyond this constraint.
analysis, I fix the total housing stock to be $\bar{H}$, which implies that the price of housing fully characterizes the state of the housing market.\(^{20}\) To focus on the use of housing as a collateral asset, I assume that saver demand is fixed at $h_{s,t} = \bar{H}_s$, so that a borrower is always the marginal buyer of housing.\(^{21}\) Finally, as is standard in the US, each loan is linked to a specific house, so that only prepaying households can adjust their housing holdings.

### 3.3 Taxation

Both types are subject to proportional taxation of labor income at rate $\tau$, which is returned in lump sum transfers $T_{b,t}$ and $T_{s,t}$ equal to the amount paid by that type. Borrower interest payments, defined as $(q_{i,t-1} - \nu) m_{i,t-1}$, are tax deductible.

### 3.4 Representative Borrower’s Problem

I show in the appendix that this individual borrower’s problem aggregates to the problem of a single representative borrower. The endogenous state variables for the representative borrower’s problem are the total start-of-period debt balance $m_{t-1}$, the total promised payment on existing debt $x_{t-1} \equiv q_{t-1} m_{t-1}$, and total start-of-period borrower housing $h_{b,t-1}$. If we define $\rho_t = \Gamma_k(\bar{r}_t)$ to be the fraction of loans prepaid, then the laws of motion for these state variables are defined by

\begin{align}
    m_t &= \rho_t m^*_t + (1 - \rho_t)(1 - \nu) \pi^{-1}_t m_{t-1} \\
    x_t &= \rho_t q^*_t m^*_t + (1 - \rho_t)(1 - \nu) \pi^{-1}_t x_{t-1} \\
    h_{b,t} &= \rho_t h^*_b + (1 - \rho_t) h_{b,t-1}
\end{align}

The representative borrower chooses consumption $c_{b,t}$, labor supply $n_{b,t}$, the size of newly purchased houses $h^*_b$, the face value of newly issued mortgages $m^*_t$, and the

\(^{20}\)The assumption that the housing stock is fixed abstracts from the important role played by residential investment in the economy, and implies that house price responses are likely overstated. But from the perspective of credit growth, the key variable is total collateral value: the product of price and quantity. Under a flexible housing supply, smaller movements in price are compensated by larger movements in quantity, leading to similar overall effects. Finally, my numerical results focus on price-rent ratios, which should not be strongly affected by this choice.

\(^{21}\)This assumption is useful under divisible housing to prevent excessive flows of housing between the two groups, which would otherwise occur unrealistically along the intensive margin of house size.
fraction of loans to prepay $\rho_t$, to maximize (1) using the aggregate utility function

$$u(c_{b,t}, h_{b,t-1}, n_{b,t}) = \log(c_{b,t}/\chi_b) + \xi \log(h_{b,t-1}/\chi_b) - \eta \left(\frac{n_{b,t}}{\chi_b}\right)^{1+\varphi}$$

subject to the budget constraint

$$c_{b,t} \leq \left(1 - \tau\right)w_t n_{b,t} - \pi_{t-1}^{-1}\left((1 - \tau)x_{t-1} + \tau v m_{t-1}\right) + \rho_t \left(m^*_t - (1 - \nu)\pi_{t-1}^{-1} m_{t-1}\right)$$

the debt constraint

$$m^*_t \leq \bar{m}_t = \bar{m}_{pti}^{li} \int_{\bar{e}_t}^{\bar{e}_t} r_t \, d\Gamma_c(e_t) + \bar{m}_{li}^{pti} (1 - \Gamma_c(\bar{e}_t)).$$

(6)

and the laws of motion (3) - (5), where

$$\bar{m}_{pti}^{li} = \theta^{li} p_{t}^bh_{b,t}^* \quad \quad m_{pti}^{li} = \frac{\theta^{pti} - \omega}{q^*_t + \alpha} \quad \quad \bar{e}_t = \bar{m}_{li}^{pti} / \bar{m}_{pti}^{li}$$

are the population average LTV and PTI limits, $\bar{e}_t$ is the threshold value of the income shock $e_{i,t}$ so that for $e_{i,t} < \bar{e}_t$, borrowers are constrained by PTI,

$$\Psi(\rho_t) = \int_{-\Gamma_c^{-1}(\rho_t)}^{\Gamma_c^{-1}(\rho_t)} \kappa d\Gamma_k(k)$$

is the average transaction cost per unit of issued debt, and $\bar{\Psi}_t$ is a proportional rebate that returns these transaction costs to borrowers.\(^{22}\)

\section*{3.5 Representative Saver’s Problem}

Just as in the borrower case, the individual saver’s problem aggregates to the problem of a representative saver. The representative saver chooses consumption $c_{s,t}$, labor supply $n_{s,t}$, and the face value of newly issued mortgages $m^*_t$ to maximize (1) using

\(^{22}\)I choose to rebate these costs to borrowers, as they likely stand in for non-monetary frictions such as inertia.
the utility function

\[ u(c_{s,t}, n_{s,t}) = \log\left(\frac{c_{s,t}}{\chi_s}\right) + \zeta \log\left(\frac{\bar{H}_s}{\chi_s}\right) - \eta \frac{(n_{s,t}/\chi_s)^{1+\phi}}{1+\phi} \]

subject to the budget constraint

\[ c_{s,t} \leq \Pi_t + (1 - \tau)w_t n_{s,t} - p_t(m^*_t - (1 - v)\pi_t^{-1}m_{t-1}) + \pi_t^{-1}x_{t-1} \]

- \delta p_t^h \bar{H}_s - R_t^{-1}b_t + b_{t-1} + T_{s,t} 

and the laws of motion (3), (4), where \( \Pi_t \) are intermediate firm profits.

### 3.6 Productive Technology

The production side of the economy is populated by a competitive final good producer and a continuum of intermediate goods producers owned by the saver. The final good producer solves the static problem

\[
\max_{P_t} \left[ \int y_{t}(i)^{\lambda - 1} di \right]^{\frac{1}{\lambda - 1}} - \int P_t(i)y_t(i) di
\]

where each input \( y_t(i) \) is purchased from an intermediate good producer at price \( P_t(i) \), and \( P_t \) is the price of the final good.

The producer of intermediate good \( i \) chooses price \( P_t(i) \) and operates the linear production function

\[ y_t(i) = a_t n_t(i) \]

to meet the final good producer’s demand, where \( n_t(i) \) is labor hours and \( a_t \) is total factor productivity, which evolves according to

\[ \log a_{t+1} = (1 - \phi_a)\mu_a + \phi_a \log a_t + \varepsilon_{a,t+1}, \quad \varepsilon_{a,t} \sim N(0, \sigma^2_a). \]

Intermediate good producers are subject to price stickiness of the Calvo-Yun form with indexation. Specifically, a fraction \( 1 - \zeta \) of firms are able to adjust their price each period, while the remaining fraction \( \zeta \) update their existing price by the rate of steady state inflation.
3.7 Monetary Authority

The monetary authority follows a Taylor rule, similar to that of Smets and Wouters (2007), of the form

$$\log R_t = \log \bar{\pi}_t + \phi_r (\log R_{t-1} - \log \bar{\pi}_{t-1})$$

$$+ (1 - \phi_r) \left[ (\log R_{ss} - \log \pi_{ss}) + \psi_\pi (\log \pi_t - \log \bar{\pi}_t) \right]$$

(7)

where the subscript “ss” refers to steady state values, where $\bar{\pi}_t$ is a time-varying inflation target defined by

$$\log \bar{\pi}_t = (1 - \psi_\pi) \log \pi_{ss} + \psi_\pi \log \bar{\pi}_{t-1} + \epsilon_{\pi,t}, \quad \epsilon_{\pi,t} \sim N(0, \sigma_{\pi}^2).$$

These shocks to the inflation target are near-permanent shocks to monetary policy that, as in Garriga et al. (2015), can be interpreted as “level factor” shocks that shift the entire term structure of nominal interest rates. In the simple bond-pricing environment of this paper, with no important source of term premia or risk premia, these shifts in long-run inflation expectations are needed for monetary policy to move long rates, but it should be kept in mind that movements in these premia would also activate the mortgage credit channel. In the limit $\psi_\pi \to \infty$, the rule (7) collapses to

$$\pi_t = \bar{\pi}_t$$

(8)

corresponding to the case of perfect inflation stabilization, which implicitly defines the value of $R_t$ needed to attain equality.

3.8 Equilibrium

A competitive equilibrium in this model is defined as a sequence of endogenous states $(m_{t-1}, x_{t-1})$, allocations $(c_{j,t}, n_{j,t})$, mortgage and housing market quantities $(h_{b,t}^*, m_t^*, \rho_t)$, and prices $(\pi_t, w_t, p_t^h, R_t, q_t^*)$ that satisfy borrower, saver, and firm optimality, and the market clearing conditions:

- **Resources:** $c_{b,t} + c_{s,t} + \delta p_t^h \bar{H} = y_t$
- **Bonds:** $b_{s,t} = 0$
- **Housing:** $h_{b,t} + \bar{H}_s = \bar{H}$. 

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3.9 Model Solution

In this section, I present two optimality conditions that summarize the main innovations of the model: simultaneous LTV and PTI constraints, and long-term debt with endogenous prepayment. The remaining optimality conditions can be found in the appendix.

The influence of the constraint structure appears most strongly in the borrower’s first order condition for housing, which requires the equilibrium house price to satisfy

\[
p_t^h = \frac{\mathbb{E}_t \left\{ u_{h,t+1}^b / u_{c,t+1}^e + \Lambda_{b,t+1} p_{t+1}^b \left[ 1 - \delta - (1 - \rho_{t+1}) C_{t+1} \right] \right\}}{1 - C_t}
\]

where \( C_t = \mu_t F_{ltv}^{t} \), \( \mu_t \) is the multiplier on the borrowing constraint, and \( F_{ltv} = 1 - \Gamma_e(\bar{e}_t) \) is the fraction of new borrowers constrained by LTV. The term \( C_t \) is the marginal collateral value of housing: the benefit to the borrower from the relaxation in her borrowing constraint due to an additional dollar of housing. Division by \( 1 - C_t \) reflects a collateral premium for housing, raising the price of housing when collateral demand is high.\(^{23}\)

In a model with an LTV constraint only, \( C_t \) would equal \( \mu_t \theta_{ltv} \), the product of the amount by which the constraint is relaxed (\( \theta_{ltv} \)) and the rate at which the borrower values the relaxation (\( \mu_t \)). But since the debt limits of PTI-constrained borrowers are not altered by an additional unit of housing, only LTV-constrained households actually receive this collateral benefit, leading to the scaling by \( F_{ltv} \). As a result, any macroeconomic forces that shift the fraction of borrowers who are LTV-constrained will also influence collateral values, translating into movements in prices. I call this mechanism — through which changes in which constraint is binding for borrowers translate into movements in house prices — the constraint switching effect.

The influence of long-term prepayable debt can be seen in the optimality condition

\(^{23}\)In contrast, the appearance of \( C_{t+1} \) in the numerator of (3.9) occurs because, with probability \( 1 - \rho_{t+1} \), the borrower will not prepay her loan. In these states of the world, the borrower will not use her housing holdings to collateralize a new loan, and does not receive the collateral benefit of housing.
for prepayment, which sets the fraction of prepaid loans to

\[
\rho_t = \Gamma_\kappa \left( (1 - \Omega_{m,b,t}) \left( 1 - \frac{(1 - \nu) \pi_t^{-1} m_{t-1}}{m_t^*} \right) \right)
\]

\[
- \Omega_{x,b,t} \left( q_t^* - q_{t-1} \frac{(1 - \nu) \pi_t^{-1} m_{t-1}}{m_t^*} \right)
\]

where \( \Omega_{m,b,t} \) and \( \Omega_{x,b,t} \) are the marginal continuation costs to the borrower of an additional unit of face-value debt, and of promised payment, respectively (see appendix). The term inside the c.d.f. \( \Gamma_\kappa \) represents the marginal benefit to prepaying an additional unit of debt, which can be decomposed into two terms reflecting borrowers’ motivations to prepay. The first term represents the incentive to take on new debt: the product of the net benefit of an additional dollar of debt ($\Omega_{m,b,t}$) and the net increase in debt per dollar of face value, since a fraction of the new loan must go to prepaying the old debt. The second term reflects the borrower’s interest rate incentive: under fixed-rate debt, prepayment is more beneficial when the interest rate on new debt \( (q_t^*) \) is low relative to the rate on existing debt \( (q_{t-1}) \). These forces will drive the frontloading effect in Section 5.2 that is key to transmission into output.

4 Calibration

The calibrated parameter values are presented in Table 1. While many parameters can be set to standard values, given the wealth of previous work on New Keynesian DSGE models, several parameters relate to features that are new to the literature, and are calibrated directly to microdata.

For the income shock distribution \( \Gamma_{e \epsilon} \), I parameterize the distribution to be lognormal, with \( \log e_{i,t} \sim N \left( -\sigma_e^2 / 2, \sigma_e^2 \right) \), which implies

\[
\int e_i d\Gamma_{e \epsilon}(e_i) = \Phi \left( \frac{\log e_t - \sigma_e^2 / 2}{\sigma_e} \right)
\]

facilitating the computation of (6). In reality, unlike in the model, borrowers may differ both in their incomes and in the size of the house that they purchase. As a result, I set \( \sigma_e \) to match the standard deviation of log house value-income ratios for new borrowers.
Table 1: Parameter Values: Baseline Calibration

<table>
<thead>
<tr>
<th>Parameter</th>
<th>Name</th>
<th>Value</th>
<th>Internal</th>
<th>Target/Source</th>
</tr>
</thead>
<tbody>
<tr>
<td><strong>Demographics and Preferences</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Fraction of borrowers</td>
<td>$x_b$</td>
<td>0.319</td>
<td>N</td>
<td>1998 SCF</td>
</tr>
<tr>
<td>Income dispersion</td>
<td>$\sigma_e$</td>
<td>0.411</td>
<td>N</td>
<td>Fannie Mae</td>
</tr>
<tr>
<td>Borrower discount factor</td>
<td>$\beta_b$</td>
<td>0.95</td>
<td>N</td>
<td>Standard</td>
</tr>
<tr>
<td>Saver discount factor</td>
<td>$\beta_s$</td>
<td>0.993</td>
<td>Y</td>
<td>$R_{ss} / \pi_{ss} = 1.03$ (ann.)</td>
</tr>
<tr>
<td>Borrower housing preference</td>
<td>$\xi$</td>
<td>0.300</td>
<td>Y</td>
<td>1998 SCF</td>
</tr>
<tr>
<td>Disutility of labor scale</td>
<td>$\eta$</td>
<td>6.351</td>
<td>Y</td>
<td>$\eta_{ss} = 1/3$</td>
</tr>
<tr>
<td>Inv. Frisch elasticity</td>
<td>$\psi$</td>
<td>1.0</td>
<td>N</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>Housing and Mortgages</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Mortgage amortization</td>
<td>$\nu$</td>
<td>1/120</td>
<td>N</td>
<td>30-year duration</td>
</tr>
<tr>
<td>Tax rate</td>
<td>$\tau$</td>
<td>0.204</td>
<td>N</td>
<td>Elenev et al. (2016)</td>
</tr>
<tr>
<td>Max PTI ratio</td>
<td>$\rho^{\text{PTI}}$</td>
<td>0.36</td>
<td>N</td>
<td>See text</td>
</tr>
<tr>
<td>Max LTV ratio</td>
<td>$\rho^{\text{LTV}}$</td>
<td>0.85</td>
<td>N</td>
<td>See text</td>
</tr>
<tr>
<td>Issuance cost mean</td>
<td>$\mu_k$</td>
<td>0.183</td>
<td>Y</td>
<td>$\rho_{ss} = 4.5%$</td>
</tr>
<tr>
<td>Issuance cost scale</td>
<td>$\kappa$</td>
<td>0.026</td>
<td>Y</td>
<td>See text</td>
</tr>
<tr>
<td>PTI offset (taxes, etc.)</td>
<td>$\alpha$</td>
<td>0.005</td>
<td>Y</td>
<td>$\eta_{ss}^* + \alpha = 10.6%$ (ann.)</td>
</tr>
<tr>
<td>PTI offset (other debt)</td>
<td>$\omega$</td>
<td>0.08</td>
<td>N</td>
<td>See text</td>
</tr>
<tr>
<td>Log housing stock</td>
<td>$\log \bar{H}$</td>
<td>2.472</td>
<td>Y</td>
<td>See text</td>
</tr>
<tr>
<td>Log saver housing stock</td>
<td>$\log \bar{H}_s$</td>
<td>2.088</td>
<td>Y</td>
<td>See text</td>
</tr>
<tr>
<td><strong>Productive Technology</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Productivity (mean)</td>
<td>$\mu_a$</td>
<td>1.099</td>
<td>Y</td>
<td>$y_{ss} = 1$</td>
</tr>
<tr>
<td>Productivity (pers.)</td>
<td>$\phi_a$</td>
<td>0.9641</td>
<td>N</td>
<td>Garriga et al. (2015)</td>
</tr>
<tr>
<td>Variety elasticity</td>
<td>$\lambda$</td>
<td>6.0</td>
<td>N</td>
<td>Standard</td>
</tr>
<tr>
<td>Price stickiness</td>
<td>$\zeta$</td>
<td>0.75</td>
<td>N</td>
<td>Standard</td>
</tr>
<tr>
<td><strong>Monetary Policy</strong></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Steady state inflation</td>
<td>$\pi_{ss}$</td>
<td>1.0075</td>
<td>N</td>
<td>$\pi_{ss} = 1.03$ (ann.)</td>
</tr>
<tr>
<td>Taylor rule (inflation)</td>
<td>$\psi_{\pi}$</td>
<td>1.5</td>
<td>N</td>
<td>Standard</td>
</tr>
<tr>
<td>Taylor rule (smoothing)</td>
<td>$\phi_r$</td>
<td>0.89</td>
<td>N</td>
<td>Campbell et al. (2014)</td>
</tr>
<tr>
<td>Trend infl (pers.)</td>
<td>$\phi_{\pi}$</td>
<td>0.994</td>
<td>N</td>
<td>Garriga et al. (2015)</td>
</tr>
</tbody>
</table>
in loan-level data from Fannie Mae, averaged over all quarters from 2000 to 2014.\textsuperscript{24}

I calibrate the fraction of borrowers $\chi_b$ and the housing preference parameter $\xi$ to match moments from the 1998 Survey of Consumer Finances. I classify borrower households in the data to be those with a house and mortgage, but less than two months’ income in liquid assets, corresponding to $\chi_b = 0.319$.\textsuperscript{25} I calibrate the housing preference parameter $\xi$ to 0.300, so that the steady state ratio of borrower house value to income, $p^h_{t}h_{b,t}/w_{t}n_{b,t}$, matches the 1998 SCF (8.89 quarterly).

Next, I calibrate the prepayment cost distribution to match Fannie Mae MBS prepayment data. The first step is to choose a functional form for $\Gamma_\kappa$. In the data, the fraction of loans prepaid in a single quarter varies from a minimum of 1.0% to a maximum of 20.8%, despite a wide range of interest rate and housing market conditions. With an upper bound so far below unity, the fit is improved by choosing $\Gamma_\kappa$ to be a mixture, such that with 1/4 probability, $\kappa$ is drawn from a logistic distribution, and with 3/4 probability, $\kappa = \infty$, in which case borrowers never prepay, delivering

$$
\Gamma_\kappa(\kappa) = \frac{1}{4} \cdot \frac{1}{1 + \exp \left( -\frac{\kappa - \mu_\kappa}{s_\kappa} \right)}.
$$

This functional form is parameterized by a location parameter $\mu_\kappa$ and a scale parameter $s_\kappa$. For a given value of $s_\kappa$, the parameter $\mu_\kappa$ is chosen to match the mean prepayment rate on fixed-rate mortgages over the sample 1994-2015 (source: eMBS).\textsuperscript{26}

For the parameter $s_\kappa$, I run a prepayment regression

$$
\logit(cpr_{i,t}) = \gamma_{0,t} + \gamma_1(q^*_t - q_{i,t-1}) + e_{i,t}
$$

using monthly MBS data from 1994-2015 with a wide range of coupon bins at each point in time, where $i$ varies across coupon bins, $cpr_{i,t}$ is the annualized prepayment rate, $q^*_t$ is the weighted average coupon rate on newly issued MBS, and $q_{i,t-1}$ is the

\textsuperscript{24}Results using loan-level data from Freddie Mac are nearly identical.

\textsuperscript{25}Although 45.3% of those households that hold more than two months’ liquid assets also hold a mortgage in the data, I still categorize them as savers as they do not appear to be liquidity-constrained, and therefore should not be sensitive to changes in their debt limits or transitory changes to income. In the model, savers can trade mortgages (and any other financial contracts) within the saver family. A small fraction of borrowers have home equity lines of credit and may not be credit constrained; excluding these households would yield a borrower share of 0.286.

\textsuperscript{26}While $\rho_t$ in the model is the rate at which borrowers prepay for the purpose of extracting equity, the data includes prepayments by the entire population (both “borrowers” and “savers”) as well as rate refinances, which change the interest rate, but not the balance on the loan. The assumption for this calibration is that the rate at which borrowers prepay to extract equity is the same as the total population prepayment rate.
weighted average coupon rate on loans in the bin at the start of the period. By incorporating the time dummies $\gamma_{0,t}$, I am able to control for variation in aggregate economic conditions, so that $\gamma_1$ is identified only from cross-sectional variation over existing coupon rates within the same period.

For the model equivalent, applying the logistic assumption for $\Gamma_\kappa$ and rearranging (9) yields

$$\text{logit}(\tilde{cpr}_t) = \gamma_{0,t} - \frac{\Omega_x^\kappa b_t}{s_\kappa} \left( q^*_t - q_{t-1} - (1 - \nu)\pi_t^{-1}m_{t-1} \right)$$  \hspace{1cm} (11)

where $\tilde{cpr}_t = 4\rho_t$ is the approximate annualized prepayment rate, and where $\gamma_{0,t}$ captures all terms not depending on $q^*_t$ or $q_{t-1}$. Given the symmetry between (10) and (11), I calibrate $s_\kappa$ so that at steady state we have $\Omega_x^\kappa / s_\kappa = \hat{\gamma}_1$, matching the sensitivities of prepayment to interest rate incentives in the model and in the regression. This procedure yields the values $s_\kappa = 0.026$ and $\mu_\kappa = 0.183$.

For the LTV limit, $\theta_{ltv} = 0.85$ is close to the mean LTV at origination over the sample, and is chosen as a compromise between the mass constrained at 80%, and the masses constrained at higher institutional limits such as 90% or 95%. For the PTI limit, I choose $\theta_{pti} = 0.36$ to match the pre-boom standard and $\omega = 0.08$ to match the traditional PTI limit excluding other debt (0.28). It is worth noting, however, that since the recent housing crash, the main constraint on new loans appears to be not 36% but 45%, while going forward, the relevant ratio is likely to be the Dodd-Frank limit of 43%. Results using this value are similar, and can be found in the appendix.

I calibrate the offset term $\alpha$ in the PTI constraint so that $q^*_t + \alpha$ is equal to 10.6% (annualized) at steady state, which is the interest and principal payment on a loan with an 8% interest rate (typical in the mid-1990s) under the exact amortization scheme for a fixed-rate mortgage, plus 1.75% annually for taxes and insurance. Since the simpler geometrically decaying coupons in the model apply too much principal repayment at the start of the loan, this calibration ensures that the higher initial payments do not imply unrealistically tight PTI limits.

For the remaining parameters, I set $\beta_s = 0.993$ and $\pi_{ss} = 1.0075$ so that steady state real rates and inflation rates are each 3%, and set $\beta_b = 0.95$. I set the tax rate $\tau$ follow-
ing Elenev et al. (2016) to the national average prior to mortgage interest deductions. To calibrate the exogenous processes for productivity \( a_t \) and the inflation target \( \pi_t \), I follow Garriga et al. (2015), who also study the impact of these shocks on long-term mortgage rates. Finally, I calibrate the housing stock and saver housing demand so that the price of housing is unity at steady state, and so the savers’ fixed housing demand is equal to the amount they would choose at steady state if they were allowed to freely select their housing holdings.\(^{30}\)

5 Results: Interest Rate Transmission

This section presents numerical results illustrating how the novel features of the model amplify transmission from nominal interest rates into debt, house prices, and economic activity, and demonstrates the implications for monetary policy. The quantitative results in this section are obtained by linearizing the model around the deterministic steady state and calculating impulse responses to the model’s fundamental shocks.

5.1 The Constraint Switching Effect

For the first main result, I find that the addition of the PTI constraint alongside the LTV constraint generates powerful transmission from interest rates into debt and house prices. To isolate the effects of the credit limit structure, I compare the model as described to this point — hereafter the Benchmark Economy — with two alternative economies: the PTI Economy which imposes only the PTI constraint \( \bar{m}_t = \bar{m}^{pti}_t \) and the LTV Economy which imposes only the LTV constraint \( \bar{m}_t = \bar{m}^{ltv}_t \). These economies are otherwise identical in their specification and parameter values, with the exception that the credit limit parameters \( \theta^{ltv} \) and \( \theta^{pti} \) are recalibrated in the PTI and LTV Economies so that their steady state debt limits match those of the Benchmark Economy.\(^{31}\)

Figure 3 displays the response to a near-permanent -1% (annualized) shock to the inflation target, which induces a similar-sized fall in long-term nominal interest rates. The first panel shows that the three economies differ widely in their debt responses to

\[ p^h_t = \frac{u^h_{s,t}}{u^c_{s,t}} + (1 - \delta) \mathbb{E}_t \left[ \Lambda_{s,t+1} p^h_{t+1} \right]. \]

\(^{30}\)The saver’s implied optimality condition for housing is

\[^{31}\)The required values are \( \theta^{ltv} = 0.714 \) and \( \theta^{pti} = 0.273 \), respectively.
the shock. To begin, the PTI Economy displays a much larger increase of debt than the LTV Economy, with more than triple the increase after 20Q (9.9% vs. 3.1%). This occurs because PTI limits are highly sensitive to changes in interest rates, since interest rates directly enter the constraint, with an elasticity near 8. In contrast, LTV constraints are not directly affected by interest rates, and display a modest response driven mostly by borrowers prepaying to lower the interest rates on their mortgages.

Turning to the Benchmark Economy, we observe a substantial increase in debt that, perhaps surprisingly, is closer to that of the PTI Economy than that of the LTV Economy. This occurs despite the fact that in the model, as is typically found in the data, the majority of borrowers are constrained by LTV (73% at steady state). This makes clear that the Benchmark Economy is not simply a convex combination of the LTV and PTI Economies, but displays qualitatively different behavior due to the constraint switching effect described in Section 3.9. As PTI limits loosen in the Benchmark Economy, many borrowers formerly constrained by PTI now find LTV to be more restrictive, driving up $F^{l tv}$ by more than three percentage points. These borrowers are now able to increase their borrowing limit by obtaining additional housing collateral, boosting housing demand. As a result, the implied price-rent ratio, defined as $p^h_t / \left( u^h_{b,t} / u^c_{b,t} \right)$, rises by more than 4% in the Benchmark Economy, compared to a small or zero rise in the LTV and PTI Economies.

The constraint switching effect not only provides a novel transmission mecha-
nism into house prices, but is also key to the Benchmark Economy’s amplified debt response. While debt limits are directly increased for PTI-constrained households, there are too few of these households to generate a large aggregate impact from this response alone. But because higher house prices increase collateral values, LTV constraints are relaxed to a much greater extent in the Benchmark Economy than in the LTV Economy. It is in fact this strong debt response of the LTV-constrained households — the majority of the borrower population — that causes the LTV and Benchmark Economy paths to diverge.\footnote{Figure A.6 in the appendix shows a counterfactual impulse response that shuts down the constraint switching effect by holding $f^{\text{LTV}}$ fixed. In this case, the debt and price-rent response of the Benchmark Economy is small and close to that of the LTV Economy.} The interaction of the two constraints therefore creates a transmission chain from interest rates, through PTI limits, into house prices, and finally into LTV limits.

5.2 The Frontloading Effect

While the interaction of LTV and PTI limits is sufficient to generate transmission from interest rates into debt and house prices, it turns out that endogenous prepayment by borrowers is crucial for transmission into output. In this class of New Keynesian model, an increase in borrowing and consumer spending can increase output, but only if it occurs in the short run, before most intermediate firms have an opportunity to reset their prices.\footnote{While nominal rigidities are important for transmission into output, the results on transmission into house prices and debt in Section 5.1 and the boom-bust experiments of Section 6 would be similar in a flexible price model (see Figure A.7 in the appendix).} While a fall in interest rates raises debt limits immediately, under long-term debt this will not translate into an increase in debt balances or spending until borrowers prepay their existing loans and take on new ones. If borrowers always prepaid at the average rate — 4.5% of loans per quarter — most new credit issuance and spending would occur too far in the future to influence output. But when borrowers can choose when to prepay, a fall in rates can induce a wave of new debt issuance, as many borrowers choose to both lock in lower fixed rates, as well as make use of their newly higher debt limits, which have been raised due to the mechanisms of the previous section.

This immediate increase in credit growth leads to a large increase in spending on impact, amplifying the economy’s output response, a phenomenon that I call the frontloading effect. To see this mechanism in action, we can once again compare alternative economies, this time contrasting the Benchmark Economy with endogenous prepay-
Figure 4: Impulse Response to 1% Productivity Shock: Comparison of LTV (Exogenous Prepayment), Benchmark (Exogenous Prepayment), and Benchmark (Endogenous Prepayment) Economies

Note: A value of 1 represents a 1% increase relative to steady state, except for “New Issuance,” $\rho_t(m_t^\ast - (1 - \nu)\pi_t^{-1}m_{t-1})$, which is measured as a percentage of steady state output (both quarterly).

ment rates determined by (9) with alternative versions of the Benchmark and LTV Economies in which $\rho_t$ is fixed to equal its steady state value $\rho_{ss}$ at all times.

To demonstrate how the frontloading effect can amplify typical business cycle fluctuations, Figure 4 shows the response to a 1% increase in productivity. This shock is deflationary, causing nominal rates to fall although real rates rise. Due to the constraint switching effect, the fall in nominal rates leads to much larger increases in debt limits in both versions of the Benchmark Economy relative to the LTV Economy.

But despite a similar rise in debt limits across the variations of the Benchmark Economy, the paths of credit issuance are sharply different. The endogenous prepayment Benchmark Economy delivers a much more frontloaded path of issuance, beginning far above and eventually falling below the smaller but more persistent issuance of the exogenous prepayment economies. This pattern leads to highly disparate effects on output, whose response is 59% larger on impact in the endogenous prepayment Benchmark Economy relative to the exogenous prepayment LTV Economy. In contrast, the two exogenous prepayment economy responses are indistinguishable, despite much larger total debt issuance in the Benchmark Economy. These results suggest that borrower prepayment is of primary importance for the effects of fluctuations in nominal rates on output.\textsuperscript{36}

A natural question in light of this finding is whether it is the fall in interest payments, or the issuance of new credit, that causes prepayment to influence demand so

\textsuperscript{36}These findings complement those of Wong (2015), who obtains a similar result in a partial equilibrium heterogeneous agent setting.
strongly. Despite potentially large redistributions between borrowers and savers following prepayment, and an extreme difference in marginal propensities to consume between the two types, it turns out that the change in payments contributes almost nothing to the output response, which is instead driven entirely by credit growth.\footnote{Figure A.8 shows that a counterfactual impulse response removing the effect of prepayment on interest rates delivers identical output responses.} The logic is similar to the analysis of the frontloading effect: while borrowers’ interest savings may be large in present value, most of the lower payments occur far in the future, where they have little influence on output.\footnote{When borrowers are expected to keep their loans for many years before prepaying, for example when they have locked in extremely low interest rates, or when the mortgage was specially modified under the Home Affordable Refinance Program, there is an additional dampening effect. In these cases, the change in payments is similar to a permanent income shock, inducing a large offsetting consumption response by the saver.} In contrast, newly issued credit can be spent immediately upon receipt, with much larger stimulatory effects.

5.3 Monetary Policy

These results on interest rate transmission have important implications for monetary policy. Specifically, I find that monetary policy can more easily stabilize inflation due to the mortgage credit channel, but contributes to larger swings in credit markets, posing a potential trade-off for policymakers. To demonstrate this, results in this section use the alternative policy rule (8), under which the central bank moves the policy rate as much as needed to perfectly stabilize inflation. While not as empirically realistic as (7), this rule provides a natural benchmark for evaluating the strength of the monetary authority: the less the policy rate must move to keep inflation at target following a shock, the more effective is monetary policy.

Figure 5 compares the response to a 1% productivity shock under the Benchmark Economy, and the exogenous prepayment LTV Economy, to demonstrate the combined contribution of the model’s novel features. In the LTV Economy, the policy rate must fall substantially on impact in order to stabilize inflation. However, in the Benchmark Economy, the policy rate \textit{rises} slightly on impact, and remains well above the LTV Economy rate for nearly 20Q. In the Benchmark Economy, a fall in rates triggers a wave of new borrowing, pushing up demand and putting upward pressure on prices, requiring less monetary stimulus to correct the deflationary shock. Since credit issuance is determined by long rates, this occurs through the \textit{expectation} of future policy rate cuts, although short-term interest rates rise slightly on impact.

Overall, these results indicate that monetary policy is stronger due to the mortgage
credit channel, requiring smaller and more gradual movements in the policy rate to stabilize inflation. But importantly, these smaller movements in the policy rate are associated with larger movements in mortgage issuance. If policymakers are concerned with the stability of credit growth as well as inflation, these dynamics may present a difficult dilemma. For an important example, consider the position of the Federal Reserve in the early 2000s, which chose to cut rates during a massive expansion of mortgage credit. Taylor (2007) has blamed this decision for the ensuing housing boom and bust, while Bernanke (2010) has argued that this action was appropriate given deflationary concerns. The preceding analysis suggests that both arguments may have merit, as there may have been no way to stabilize inflation without further destabilizing credit markets. These results therefore provide a potential rationale for imperfect inflation stabilization, or for the use of instruments other than monetary policy to influence credit markets.

6 Results: Credit Standards and the Boom

The previous section focused on the model dynamics under a single credit regime, with $\theta^{ltv}$ and $\theta^{pti}$ fixed, as these maximum ratios are typically stable at business cycle frequencies. But credit standards can change over time, and did so dramatically during the recent boom-bust episode. In this section, I present several experiments varying credit conditions to examine the implications of the model for the sources of the boom-bust, and for the type of macroprudential policy that might have limited its
Table 2: Results: Boom Experiments

<table>
<thead>
<tr>
<th>Experiment</th>
<th>Price-Rent (Of Actual)</th>
<th>Debt-Income (Of Actual)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Data</td>
<td>60%</td>
<td>56%</td>
</tr>
<tr>
<td><strong>Credit Liberalization Experiments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTV Liberalized</td>
<td>-2% (-3%)</td>
<td>11% (20%)</td>
</tr>
<tr>
<td>PTI Liberalized</td>
<td>26% (42%)</td>
<td>29% (51%)</td>
</tr>
<tr>
<td>Both Liberalized</td>
<td>31% (51%)</td>
<td>53% (95%)</td>
</tr>
<tr>
<td>Dodd-Frank (Appendix)</td>
<td>12% (20%)</td>
<td>29% (52%)</td>
</tr>
<tr>
<td><strong>House Price Expectations Experiments</strong></td>
<td></td>
<td></td>
</tr>
<tr>
<td>LTV Economy</td>
<td>61% (101%)</td>
<td>52% (93%)</td>
</tr>
<tr>
<td>Benchmark Economy</td>
<td>25% (41%)</td>
<td>12% (22%)</td>
</tr>
</tbody>
</table>

Note: Table corresponds to Figures 6, 7, and A.11 (appendix). For each experiment, “Price-Rent” and “Debt-Income” columns denote the rise from the start of the experiment to the peak of the boom, 32Q later, for price-rent and debt-household income ratios, respectively. The columns “(Of Actual)” denote the fraction of the observed increase of this variable in the data explained by this path.

To simulate a hypothetical boom-bust, the experiments trace out nonlinear transition paths in a deterministic version of the Benchmark Economy after a surprise announcement that $\theta_{ltv}$ or $\theta_{pti}$ has changed permanently, followed by a subsequent surprise announcement that parameters have reverted to their baseline values. The time gap between the announcements is 32Q, corresponding to the duration of the boom in price-rent ratios (1997 Q4 - 2006 Q1). The results of these experiments are reported in Table 2, and are further analyzed below. For magnitudes, I compare the resulting rise in implied price-rent ratios $p_{t}^{h}/(u_{h,b,t}^{u}/u_{b,t}^{c})$ and debt-household income ratios $m_{t}/y_{t}$ to their counterparts in the data, which increased by 60% and 56% over this period, respectively.

Figure 6 shows the responses to changing credit standards. To begin, the LTV Liberalized experiment increases $\theta_{ltv}$ from 0.85 to 0.99, followed by a reversal. Although a relaxation of LTV standards is often proposed as a candidate cause of the boom, this event alone cannot generate a large boom when PTI limits are held at their baseline severity.

To analyze the differential effects of changing LTV and PTI standards, I treat these shifts as exogenous. However, changes in standards during the recent boom likely reflect deeper endogenous changes that altered lenders’ perceptions of credit risk — an important area for future research.

Data equivalents, plotted in Figure A.9 in the appendix, are obtained from the Federal Reserve Board of Governors, Flow of Funds. Prices are household real estate values (LM155035015.Q) while debt is household home mortgages (FL153165105.Q). Household income is disposable personal income (FA156012005.Q).

While the exact amount by which LTV limits were relaxed during the boom is unclear, this near-complete relaxation is designed to give LTV relaxation the best possible chance at explaining the boom.
values. Despite a near-complete liberalization of LTV, we observe only a small rise in debt-household income ratios, while price-rent ratios actually fall. This result is entirely due to the presence of the PTI limit, as a similar liberalization in the LTV Economy would indeed produce a dramatic boom.\textsuperscript{42}

The presence of PTI limits dampens the response to LTV liberalization for two reasons. First, there is a direct effect, since PTI-constrained borrowers cannot increase their credit balances in response to this change. But, more importantly, there is a general equilibrium response due to the constraint switching effect. As LTV limits loosen, many previously LTV-constrained borrowers now find their PTI limits to be more restrictive. The resulting fall in $F_{ltv}$ of roughly 10 percentage points depresses collateral demand and price-rent ratios. The failure of house prices to boom in turn limits the ability of LTV-constrained households to borrow, dampening the increase in debt.

Next, consider the PTI Liberalized experiment in Figure 6, which shows the response to an increase in $\theta_{pti}$ from 0.36 to 0.54 and its subsequent reversal — likely a conservative calibration of the change in standards given the evidence in Figure 2, which was further exacerbated in practice by exotic mortgage products and low-documentation loans that loosened PTI limits further.\textsuperscript{43} In sharp contrast to the LTV Liberalized case, the PTI Liberalization experiment generates a large boom, accounting for nearly half of the observed rise in price-rent and debt-household income ratios. While these results clearly leave room for other factors, they indicate an important role for changing PTI standards in driving the boom-bust cycle.

That the PTI-driven boom vastly exceeds the LTV-driven boom, despite the fact that only a minority of borrowers are PTI-constrained, is once again due to the constraint switching effect. As PTI limits have loosened, more borrowers find themselves constrained by LTV, pushing up the demand for collateral, driving up house prices, and relaxing debt limits for the LTV-constrained majority. Importantly, this pathway provides a new perspective on recent empirical research showing that borrowing during the boom increased evenly across the income spectrum, and occurred largely in response to increases in house prices.\textsuperscript{44} While the simulated boom is initiated by the relaxation of income constraints, new borrowing in the experiment is largely undertaken by LTV-constrained households responding to the rise in house prices, consistent with these empirical findings.

\textsuperscript{42}See Figure A.10 in the appendix.

\textsuperscript{43}Adjustable-rate and low-amortization/interest-only mortgages offered lower initial payments during the boom, while low-documentation loans allowed borrowers to inflate their stated income, in both cases lowering the effective PTI ratios on a given loan.

\textsuperscript{44}See e.g., Adelino et al. (2015) and Foote et al. (2016).
Moreover, while only partly sufficient to explain the boom, a relaxation of PTI standards was likely a necessary condition, enabling other forces to drive the rest of the boom. This can be seen in the Both Liberalized experiment of Figure 6, which shows the result of simultaneously relaxing \((\theta_{ltv}, \theta_{pti})\) from \((0.85, 0.36)\) to \((0.99, 0.54)\). These responses show that the two liberalizations are complements, so that an LTV liberalization has a much larger impact when PTI standards are also being relaxed. The additional rise in debt-household income ratios moving from the PTI Liberalized experiment to the Both Liberalized experiment is more than twice as large as the rise under the LTV Liberalized experiment alone. Moreover, while LTV liberalization in isolation caused price-rent ratios to fall, when both standards are loosened, price-rent ratios rise by more than under a PTI liberalization alone. These results indicate that a relaxation of LTV constraints contributes much more powerfully to booms when accompanied by a loosening of PTI standards.

While the previous results consider a simultaneous liberalization of PTI standards, the absence of PTI limits due to a previous liberalization can also amplify the contributions of other factors. To show this, Figure 7 plots the responses to an expected future increase of 70% in the housing preference parameter \(\xi\) in the Benchmark and LTV Economies, which unexpectedly does not occur. While this experiment can account for virtually the entire boom in the LTV Economy, the responses of the Benchmark Economy are seriously dampened, with less than half the rise in price-rent ratios, and less than one-fourth the rise in debt-household income ratios. In the Benchmark Economy, rising house prices caused by the expected increase in value endogenously loosen LTV constraints, causing some households to become PTI-constrained, and again acti-
Figure 7: House Price Expectations Experiments

Note: A value of 1 represents a 1% increase relative to steady state, except for $l_{ltv}$, which is measured in percentage points. At time 0, agents learn that in 32Q, the housing preference parameter $\xi$ will increase by 70%. But after 32Q, the parameter unexpectedly is not increased.

These results have important implications for macroprudential policy. As noted by Jácome and Mitra (2015), while caps on both LTV and PTI ratios are common regulatory measures around the world, there is little theoretical guidance to date about which cap should be used. Given the results above, the model clearly indicates that a cap on PTI ratios, not LTV ratios, is the more effective macroprudential policy for limiting boom-bust cycles. A cap on PTI ratios can both directly prevent booms caused by a liberalization of PTI, as well as dampen the influence of other factors that are much stronger when PTI limits are absent or simultaneously relaxed.

Of particular relevance is the recent Dodd-Frank legislation, which for the first time imposed a regulatory cap of 43% on PTI ratios for US mortgages.45 While this maximum ratio is higher than the pre-boom standard (36%), results indicate that it could have considerably dampened the recent boom. Figure A.11 in the appendix shows that an alternative experiment increasing $l_{ltv}$ to 0.99 but letting $l_{pti}$ rise only to the Dodd-Frank limit of 0.43 reduces the rise in price-rent ratios by nearly two-thirds relative to the Both Liberalized experiment.

7 Conclusion

In this paper, I developed a general equilibrium framework centered on two novel features: the combination of LTV and PTI limits, and the endogenous prepayment

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45Technically, this is not a strict limit, but a restriction on Qualified Mortgages, a class of mortgage that lenders are strongly incentivized to issue.
of long-term debt. When calibrated to US mortgage microdata, these features greatly amplify transmission from interest rates into debt, house prices, and economic activity. The effects on credit and house prices are created largely by the constraint switching effect, through which changes in which of the two constraints is binding for borrowers translate into movements in house prices. The effects on economic activity are due mainly to the frontloading effect, through which the prepayment decisions of borrowers generate waves of new borrowing and spending following a movement in interest rates. Monetary policy can more potently stabilize inflation due to these forces, but contributes to larger movements in credit growth, posing a potential trade-off for policymakers. A PTI liberalization is essential to explaining the boom-bust, and cap on PTI ratios, not LTV ratios, is the more effective macroprudential policy.

This work provides several avenues for future research. Perhaps most important, this framework abstracts from default — the primitive risk that LTV and PTI limits are designed to mitigate — precluding a serious welfare analysis of different macroprudential policies. Partially as a result, it is also silent on the deeper causes for the movements in credit standards observed in recent years, which could be endogenized in subsequent work. Finally, incorporating a richer intermediary sector that is able to realistically capture the financial crisis could allow for analysis of both actual and alternative credit market interventions during the bust.

References


A Appendix

The appendix is structured as follows. Section A.1 completes the derivation of the equilibrium conditions for the model. Section A.2 demonstrates the aggregation result. Section A.3 describes the data used in the calibration and plots. Section A.4 presents extensions of the baseline model. Supplementary tables and figures can be found at the end of the appendix.

A.1 Model Solution

This section supplements Section 3.9 by providing the set of optimality conditions for the model.

A.1.1 Borrower Optimality

Optimality of labor supply, \( n_{b,t} \), implies the intratemporal condition

\[
- \frac{u_{n_{b,t}}}{u_{c_{b,t}}} = (1 - \tau)w_t + \mu_t \rho_t \left( \frac{\theta_{pti} - \omega}{q_{b,t} + \alpha} \right) \int \tilde{e}_t d\Gamma_e(e_t),
\]

where \( \mu_t \) is the multiplier on the borrower’s aggregate credit limit, and \( \Omega_{b,t}^{m} \) and \( \Omega_{b,t}^{x} \) are the marginal continuation costs to the borrower of taking on an additional dollar of face value debt, and of promising an additional dollar of initial payments, defined by

\[
\Omega_{b,t}^{m} = \mathbb{E}_t \left\{ \Lambda_{b,t+1} \tau_{t+1}^{-1} \left[ \nu \tau + (1 - \nu) \rho_{t+1} + (1 - \nu) (1 - \rho_{t+1}) \Omega_{b,t+1}^{m} \right] \right\}
\]

\[
\Omega_{b,t}^{x} = \mathbb{E}_t \left\{ \Lambda_{b,t+1} \tau_{t+1}^{-1} \left[ (1 - \tau) + (1 - \nu) (1 - \rho_{t+1}) \Omega_{b,t+1}^{x} \right] \right\}
\]

respectively.

\[46\]Because I assume that the borrower chooses her labor supply before deciding whether to prepay, this has a very small effect on labor supply, equivalent to a 2.5% increase in wages in steady state. Results assuming that borrowers do not internalize the effect of their labor supply decision on their credit availability, which sets this term to zero, are virtually identical.
A.1.2 Saver Optimality

The saver optimality conditions are similar to those of the borrower, and are defined by

\[-\frac{u^n_s}{u^c_s} = (1 - \tau)w_t\]

\[1 = R_t \mathbb{E}_t \left[ \Lambda_{s,t+1}^{-1} \pi_{t+1}^{s-1} \right] \]

\[1 = \Omega_{s,t}^m + \Omega_{s,t}^x q_t^* \]

where \(\Omega_{s,t}^m\) and \(\Omega_{s,t}^x\) are the marginal continuation benefits to the saver of an additional unit of face value and an additional dollar of promised initial payments, respectively. These values are defined by

\[\Omega_{s,t}^m = \mathbb{E}_t \left\{ \Lambda_{s,t+1}^{-1} \left[ (1 - \nu) \rho_t + (1 - \nu)(1 - \rho_{t+1}) \Omega_{s,t+1}^m \right] \right\} \]

\[\Omega_{s,t}^x = \mathbb{E}_t \left\{ \Lambda_{s,t+1}^{-1} \left[ 1 + (1 - \nu)(1 - \rho_{t+1}) \Omega_{s,t+1}^x \right] \right\}. \]

These expressions are equivalent to the terms in the borrower’s problem, with the exception that savers are unconstrained (\(\mu = 0\)), use a different stochastic discount factor, do not optimize over housing, and have an additional optimality condition from trade in the one-period bond.

A.1.3 Intermediate and Final Good Producer Optimality

The solution to the intermediate and final good producers’ problems is standard and can be summarized by the following system of equations

\[N_t = y_t \left( \frac{mc_t}{mc_{s,ss}} \right) + \zeta \mathbb{E}_t \left[ \Lambda_{s,t+1} \left( \frac{\pi_{t+1}}{\pi_{ss}} \right)^{\lambda} N_{t+1} \right] \]

\[D_t = y_t + \zeta \mathbb{E}_t \left[ \Lambda_{s,t+1} \left( \frac{\pi_{t+1}}{\pi_{ss}} \right)^{\lambda-1} D_{t+1} \right] \]

\[\bar{p}_t = \frac{N_t}{D_t} \]

\[\pi_t = \pi_{ss} \left[ 1 - \left( \frac{1 - \zeta}{\zeta} \right)^{\lambda-1} \right]^{\frac{1}{\lambda-1}} \]

\[\Delta_t = (1 - \zeta) \bar{p}_t^{-\lambda} + \zeta (\pi_t / \pi_{ss})^\lambda \Delta_{t-1} \]

38
\[ y_t = \frac{a_t n_t}{\Delta_t} \]

where \( y_t \) is total output, \( N_t \) and \( D_t \) are auxiliary variables, \( \bar{p}_t \) is the ratio of the optimal price for resetting firms relative to the average price, and \( \Delta_t \) is price dispersion.

### A.2 Aggregation

This section demonstrates the equivalence of the representative borrower’s problem with the individual borrower’s problem. The proof of the equivalence of problems of the individual saver and representative saver is symmetric.

In the individual’s problem I assume that each borrower owns a house of a given size but can freely buy and sell housing services on an intra-borrower rental market. The individual borrower chooses consumption of nondurables \( c_{i,t} \), rental of housing services \( h_{i,rent}^{rent} \), labor supply \( n_{i,t} \), an indicator for the choice to prepay \( I_t \in \{0, 1\} \), her target owned house size \( h_{t}^* \), and mortgage size \( m_{i,t}^* \) conditional on prepayment, and a vector of Arrow securities \( a_{i,t}(s_{t+1}) \) traded among borrowers to maximize (1) subject to the budget constraint

\[
c_{i,t} \leq (1 - \tau)w_t n_{i,t} - \pi_t^{-1} x_{i,t-1} + \tau \pi_t^{-1} (x_{i,t-1} - v m_{t-1}) + \text{rent}_t(h_{i,t} - h_{i,rent}^{rent}) - \delta p^h_t h_{i,t-1} - I_t(k_{i,t}) \left[ (m_{i,t}^* - (1 - \nu) \pi_t^{-1} m_{i,t-1}) - p^h_t(h_{i,t}^* - h_{i,t-1}) - (k_{i,t} - \text{Rebate}_t) m_{i,t}^* \right] + a_{i,t-1}(s_t) + \sum_{s_{t+1}|s_t} p^a_t(s_{t+1}) a_{i,t}(s_{t+1}) + T_{b,t} \]

the debt constraint

\[
m_{i,t}^* \leq \min(m_{i,t}^{lti}, m_{i,t}^{pti}) \]

and the laws of motion

\[
m_{i,t} = I_t(k_{i,t}) m_{i,t}^* + (1 - I_t(k_{i,t})) (1 - \nu) \pi_t^{-1} m_{i,t-1} \quad (16) \]
\[
h_{i,t} = I_t(k_{i,t}) h_{i,t}^* + (1 - I_t(k_{i,t})) h_{i,t-1} \quad (17) \]
\[
x_{i,t} = I_t(k_{i,t}) q_t^* m_{i,t}^* + (1 - I_t(k_{i,t})) (1 - \nu) \pi_t^{-1} x_{i,t-1}. \quad (18) \]

The assumption that prepayment can be chosen based only on aggregate and not individual conditions, other than the draw of the transaction cost \( \kappa_{i,t} \) is expressed by the
lack of a subscript $i$ on $I_t$. This policy is chosen before time 0. The exact timing for the other controls is as follows:

1. Borrowers choose labor supply $n_{i,t}$.

2. Borrowers choose how much housing they will purchase conditional on prepayment.

3. Borrowers draw $\kappa_{i,t}$ and determine whether to prepay based on the pre-time 0 choice of $I_t(\kappa_{i,t})$.

4. Borrowers draw $e_{i,t}$.

5. Prepaying borrowers choose their new loan size $m_{i,t}^*$ subject to their credit limits.

6. Borrowers realize insurance claims, buy new Arrow securities, and choose consumption and rental housing.

The Lagrangian is given by

$$
L = \sum_{t=0}^{\infty} \sum_{s^t} \beta_b^t \int_{e_{i,t}} \int_{\kappa_{i,t}} \int \int \left\{ u(c_{i,t}, h_{i,t}^{rent}, n_{i,t})

+ \lambda_{i,t} \left[ + (1 - \tau)w_t n_{i,t} - \pi_t^{-1} x_{i,t-1} + \tau \pi_t^{-1} (x_{i,t-1} - vm_{i,t-1})

+ \text{rent}_t(h_{i,t} - h_{i,t}^{rent}) - \delta p_t h_{i,t-1}

- I_t(\kappa_{i,t}) \left( (m_{i,t}^* - (1 - v) \pi_t^{-1} m_{i,t-1}) - p_t^h (h_{i,t}^* - h_{i,t-1})

- (\kappa_{i,t} - \text{Rebate}_t) m_{i,t}^*) \right]

+ a_{i,t-1}(s_t) + \sum_{s_{t+1}|s_t} p_t^s(s_{t+1}) a_{i,t}(s_{t+1}) - c_{i,t}

+ \mu_{i,t} I_t(\kappa_{i,t}) \left( \min(\tilde{m}_{i,t}^{\text{lv}}, \tilde{m}_{i,t}^{\text{pti}}) - m_{i,t}^* \right) \right\} dF_c(e_{i,t}^t) dF_{\kappa}(\kappa_{i,t}^t) dt.
$$
where superscript $t$ implies the history from time 0 to $t$. The optimality conditions are

$$(c_{i,t}) : \quad u^e_{i,t} = \lambda_{i,t}$$

$$(a_{i,t}(s_{t+1})) : \quad p_{i}^{h}\lambda_{i,t} = \beta_b \mathbb{E}_t \lambda_{i,t+1}$$

$$(n_{i,t}) : \quad u^n_{i,t} + \lambda_{i,t}(1 - \tau)w_t \int e_{i,t} d\Gamma_c(e_{i,t})$$

$$+ \lambda_{i,t} \mu_{i,t} \int \int I_t(\kappa_{i,t}) \frac{\partial m_{i,t}^p}{\partial n_{i,t}} 1_{\{m_{i,t}^p < m_{i,t}^m\}} \ d\Gamma_c(e_{i,t}) \ d\Gamma_{x}(\kappa_{i,t}) = 0$$

$$h^{rent}_{i,t} : \quad u^h_{i,t} = \lambda_{i,t}\text{rent}_t$$

$$(h^s_{i,t}) : \quad \int \left[ \Omega^h_{i,t} - p^h_{i} + \mu_{i,t} 1_{\{\epsilon_{i,t} \geq \epsilon_t\}} \theta_{i,t} \right] d\Gamma_c(e_{i,t}) = 0$$

$$(m^s_{i,t}) : \quad \Omega^m_{i,t} + \Omega^x_{i,t} \mu_{i,t} - 1 + \mu_{i,t} = 0$$

$$\Pi_t(\kappa_{i,t}) : \quad \kappa^t_i = \int \int_{\epsilon_{i,t}^i} \left\{ (1 - \Omega^m_{i,t}) (m^*_{i,t} - (1 - \nu) \pi^{-1}_t m_{i,t-1}) - \Omega^x_{i,t} (q^*_{i} m^*_{i,t} - (1 - \nu) \pi^{-1}_t x_{i,t-1}) - (p^h_{i} - \Omega^h_{i,t}) (h^s_{i,t} - h_{i,t-1}) \right\} dF(e_{i,t}) dF(k^t_{i,t})$$

where

$$\Omega^h_{i,t} = \mathbb{E}_t \left\{ \Lambda_{i,t+1} \left[ \left( \text{rent}_{t+1} - \delta \right) + \rho_{t+1} p^h_{i+1} + (1 - \rho_{t+1}) \Omega^h_{i,t+1} \right] \right\}$$

$$\Omega^m_{i,t} = \mathbb{E}_t \left\{ \Lambda_{i,t+1} \pi^{-1}_{t+1} \left[ \nu \tau + (1 - \nu) \rho_{t+1} + (1 - \nu) (1 - \rho_{t+1}) \Omega^m_{i,t+1} \right] \right\}$$

$$\Omega^x_{i,t} = \mathbb{E}_t \left\{ \Lambda_{i,t+1} \pi^{-1}_{t+1} \left[ (1 - \tau) + (1 - \nu) (1 - \rho_{t+1}) \Omega^x_{i,t+1} \right] \right\}$$

and where $\Lambda_{i,t+1} = \beta \lambda_{i,t+1} / \lambda_{i,t}$. Note that the $\Pi_t(\kappa_{i,t})$ optimality condition follows from the threshold prepayee’s indifference toward prepaying and not prepaying. Given the assumption that the prepayment decision cannot condition on individual states, the probability of prepayment in the next period $\rho_{t+1}$ does not depend on $i$ or on other time $t$ controls.

I now demonstrate that these optimality conditions are equivalent to those derived from the representative borrower’s problem. I seek a symmetric equilibrium, in which all borrowers have equal lifetime wealth at time 0. From the $a_{i,t}(s_{t+1})$ optimality condition it follows that $\Lambda_{i,t+1}$ takes the identical value $\Lambda_{b,t+1}$ for all $i$. In the symmetric equilibrium, this implies that $\lambda_{i,t}$ is identical across all agents, and so $c_{i,t}$ is identically equal to $c_{b,t} / \lambda_b$. As a result, we immediately obtain $h^{rent}_{i,t}$ identically equal to $h_{b,t-1} / \lambda_b$ across agents.

Since all of the components of the $\Omega$ equations are identical, the $\Omega^h_{i,t}$, $\Omega^m_{i,t}$, and $\Omega^x_{i,t}$
terms are identical across all agents $i$, and the $\Omega_{i}^{m}$ and $\Omega_{i}^{x}$ terms satisfy (14) and (15). Applying this result to the $m_{i,t}^{*}$ condition, we find that the value of $\mu_{i,t}$ is identical across borrowers, yielding (13). Substituting into the $h_{i,t}^{*}$ equation we obtain

$$\Omega_{i}^{h} = (1 - \mu_{t}F_{i}^{ltv}\theta_{i}^{ltv})p_{i}^{ltv}$$

which combined with the $\Omega_{i}^{h}$ and $h_{i,t}^{rent}$ conditions yields (3.9). Applying the results above, and the equilibrium condition $h_{i,t}^{*} = h_{i,t} = h_{b,t}$ yields (9). We can also integrate the $n_{i,t}$ condition over $e_{i,t}$ and $\kappa_{i,t}$ to yield

$$-\frac{u_{i,t}^{n}}{u_{i,t}^{c}} = (1 - \tau)w_{t} + \mu_{t}p_{t} \left( \frac{\theta_{i}^{tr}w_{t}}{q_{i}^{c} + \alpha} \right) \int e_{i}^{t} d\Gamma_{e}(e_{i,t})$$

which implies $n_{i,t} = n_{b,t}/\chi_{b}$ for all $i$, and delivers (12). Finally, integrating (16) - (18) yields (3) - (4).

### A.3 Data Description

The analysis relies on three data sets, which are described below.

#### A.3.1 Fannie Mae Loan-Level Data

This set is taken from Fannie Mae’s Single Family Loan Performance Data. From the Fannie Mae data description:

The population includes a subset of Fannie Mae’s 30-year, fully amortizing, full documentation, single-family, conventional fixed-rate mortgages. This dataset does not include data on adjustable-rate mortgage loans, balloon mortgage loans, interest-only mortgage loans, mortgage loans with prepayment penalties, government-insured mortgage loans, Home Affordable Refinance Program (HARP) mortgage loans, Refi Plus mortgage loans, and non-standard mortgage loans. Certain types of mortgage loans (e.g., mortgage loans with LTVs greater than 97 percent, Alt-A, other mortgage loans with reduced documentation and/or streamlined processing, and programs or variances that are ineligible today) have been excluded in order to make the dataset more reflective of current underwriting guidelines. Also excluded are mortgage loans originated prior to 1999, sold with lender recourse or subject to other third-party risk-sharing arrangements, or were acquired by Fannie Mae on a negotiated bulk basis.

The sample contains over 21 million loans acquired from Jan, 2000 to March 2012.

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A.3.2 Freddie Mac Loan-Level Data

This set is taken from Freddie Mac’s Single Family Loan-Level Dataset. The data set contains approximately 17 million 30-year, fixed-rate mortgages originated between January 1, 1999, and September 30, 2013. Data plots corresponding to those for Fannie Mae data in the main text can be found in Figure A.1.

A.3.3 Pool-Level Agency MBS Data

This data set from eMBS contains pool-level MBS data on all Fannie Mae, Freddie Mac, and Ginnie Mae products. The data are available at monthly frequency and are disaggregated by product type (e.g., 30-Year Fixed Rate), by coupon bin (in increments of 0.25% or 0.5%), and by either production year or state. Available variables include principal balance, conditional prepayment rate, level of issuance, weighted average coupon, and weighted average time to maturity.

A.4 Extensions

This section contains two extensions to the baseline model: a specification with adjustable-rate mortgages, and a calibration with a higher PTI limit (43%) corresponding to the new limits under the Dodd-Frank Act.

A.4.1 Adjustable-Rate Mortgages

This section considers a version of the model using adjustable-rate mortgages (ARMs) instead of fixed-rate mortgages (FRMs). Under an ARM contract, the saver gives the borrower $1 at origination. In exchange, the saver receives $(1 - \nu)^k q^*_t q_{t+k-1}^* \nu$ at time $t + k$, for all $k > 0$ until prepayment, where $q^*_{t+k-1} = (R_{t+k-1} - 1) + \nu$. This coupon rate is obtained from arbitrage considerations, since a saver must be indifferent between holding an adjustable-rate mortgage for one period and the one-period bond, since both are short-term risk-free assets.

Under ARM contracts, promised payment is no longer an endogenous state variable, but is instead defined period-by-period using

$$x_t = q^*_t m_t.$$
Figure A.1: Freddie Mac Data: CLTV and PTI on Newly Originated Mortgages

Note: Histograms are weighted by loan balance. Source: Freddie Mac Single Family Loan-Level Dataset.
Correspondingly, \( \Omega_{j,t}^x \) and \( \Omega_{j,t}^m \) can be combined into a single term \( \Omega_{j,t} \), that represents the total continuation cost of an additional unit of debt. As a result, the borrower’s optimality conditions in the ARM case become

\[
\rho_t = \Gamma \kappa \left\{ (1 - \Omega_{b,t}) \left(1 - \frac{(1 - v) \pi_t^{-1} m_{t-1}}{m_t^*} \right) \right\}
\]

\[
\Omega_{b,t} = 1 - \mu_t
\]

for

\[
\Omega_{b,t} = \mathbb{E}_t \left\{ \Lambda_{b,t+1}^s \left[ (1 - \tau) q_t^* + \tau v + (1 - v) \rho_{t+1} + (1 - v)(1 - \rho_{t+1}) \Omega_{b,t+1} \right] \right\}.
\]

The saver’s optimality conditions for \( m_t^* \) in the ARM case becomes

\[
\Omega_{s,t} = 1
\]

where

\[
\Omega_{s,t} = \mathbb{E}_t \left\{ \Lambda_{s,t+1}^s \left[ q_t^* + (1 - v) \rho_{t+1} + (1 - v)(1 - \rho_{t+1}) \Omega_{s,t+1} \right] \right\}.
\]

To see the impact of the type of mortgage contract on the dynamics, we can compare the Benchmark Economy with an ARM Economy in which contracts are defined as in this section. Impulse responses for these economies to an inflation target shock can be seen in Figure A.2. While the responses are qualitatively similar, the Benchmark economy displays larger responses of debt and output, due to the relatively higher increase in prepayments in the Benchmark Economy. The reason is that a fall in long-term rates provides a larger incentive to prepay in the Benchmark Economy, where borrowers can lock in the low rate for the future, than in the ARM Economy, where interest rates are determined period-by-period.
Figure A.2: Impulse Response to -1% (Annualized) Inflation Target Shock: Comparison of Benchmark, ARM Economies

Note: A value of 1 represents a 1% increase relative to steady state, except for $F_{ltv}$ which is measured in percentage points.

A.4.2 Alternative PTI Calibration

In this section, I present results using a higher calibration for the PTI limit of 43%, corresponding to the maximum for Qualified Mortgages under the Dodd-Frank Act. Impulse responses, shown in Figure A.3, demonstrate strong effects of incorporating PTI limits alongside LTV limits, although an even smaller minority of borrowers (13%) are constrained by PTI at equilibrium.

Figure A.3: Impulse Response to -1% (Annualized) Inflation Target Shock: Comparison of LTV, PTI, Benchmark Economies, 43% (Dodd-Frank) PTI Limit

Note: A value of 1 represents a 1% increase relative to steady state, except for $F_{ltv}$ which is measured in percentage points.
## Supplementary Tables

Table A.1: Prepayment Regression

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<td>$q_{i}^* - q_{i,t-1}$</td>
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<td>(0.012)</td>
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<tr>
<td>Time FE</td>
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<td>Observations</td>
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<tr>
<td>Adj. $R^2$</td>
<td>0.663</td>
</tr>
</tbody>
</table>

*Note:* Results are from a logistic regression computing (10). Observations are Fannie Mae 30-Year MBS (FNM30) data (source: eMBS) and the sample is Jan 1994 - Jan 2015. A single observation is a pool of all mortgages with a given coupon rate, ranging from 2.0 to 17.0. The procedure is weighted least squares, where the weight for each coupon bin is the total face value of mortgages in that bin. Standard errors, displayed in parentheses, are corrected for heteroskedasticity. The data are scaled so that, e.g., 1.0 corresponds to a 1% annual coupon payment. Since the model measures coupon rates at a quarterly rate, so that a coupon of 0.0025 corresponds to a 1% annual coupon rate, the regression coefficient must be scaled the value $\gamma_1 = 284.6$ for use in the calibration exercise.
Figure A.4: Prepayment Rate vs. Interest Rate Incentive

Note: “Prepayment Rate” is the conditional prepayment rate, which is an annualized rate measuring the fraction of loans that would be prepaid if the monthly prepayment rate continued for an entire year. “Rate Incentive” is the percent spread between weighted average coupon rates on existing loans in Fannie Mae 30 Year MBS pools (FNM30), and on newly issued loans in the same pools. The value represents the approximate interest savings that a borrower would obtain by refinancing. The source for all data is eMBS.
Figure A.5: Fannie Mae: CLTV and PTI Percentiles for Newly Originated Purchase Mortgages

Note: Plots report unweighted percentiles. Source: Fannie Mae Single Family Loan Performance Dataset, issuance data.
Figure A.6: Impulse Response to -1% (Annualized) Inflation Target Shock: Comparison of LTV, Benchmark, and Fixed $F_{ltv}$ Economies

A value of 1 represents a 1% increase relative to steady state, except for $F_{ltv}$ which is measured in percentage points.

Figure A.7: Impulse Response to -1% (Annualized) Inflation Target Shock: Comparison of LTV, PTI, Benchmark Economies, Flexible Prices

Note: Results are obtained in an alternative version of the model with $\zeta = 0$, so that all intermediate goods prices are reset each period. A value of 1 represents a 1% increase relative to steady state, except for $F_{ltv}$, which is measured in percentage points.
Figure A.9: Data: Boom-Bust Period

Note: A value of 1 represents a 1% increase relative to the initial period. The sample spans the period 1998 Q1 \((t = 0)\) to 2015 Q1 \((t = 69)\). Source: Federal Reserve Board of Governors, Flow of Funds. Prices are household real estate values (LM155035015.Q) while debt is household home mortgages (FL153165105.Q). Household income is disposable personal income (FA156012005.Q).

Figure A.8: Impulse Response to 1% (Annualized) Productivity Shock, Comparison of Benchmark No Rate Change, Economies

Note: A value of 1 represents a 1% increase relative to steady state, except for “New Issuance,” \(\rho_t (m_t^* - (1 - \nu)\pi_t^{-1}m_{t-1})\), which is measured as a percentage of steady state output (both quarterly). The “No Rate Change” responses correspond to a counterfactual economy in which borrowers still prepay using the rule (9), but do not update the interest rate following prepayment, so that

\[
x_t = q_t (m_t^* - (1 - \nu)\pi_t^{-1}m_{t-1}) + (1 - \nu)\pi_t^{-1}x_{t-1}.
\]
Figure A.10: Credit Liberalization Experiment: LTV Economy

Note: A value of 1 represents a 1% increase relative to steady state. At time zero, the LTV limit $\theta^{ltv}$ is unexpectedly loosened from 70% to 80%, and after 32Q, is unexpectedly tightened from 80% to 70%.

Figure A.11: Credit Liberalization Experiment: Both Liberalized vs. Dodd-Frank Limit

Note: A value of 1 represents a 1% increase relative to steady state, except for $F^{ltv}$, which is measured in percentage points. For the path “Dodd-Frank”: at time zero, the LTV limit $\theta^{ltv}$ and PTI limit $\theta^{pti}$ are both unexpectedly loosened from (0.85, 0.36) to (0.99, 0.43), and after 32Q, is unexpectedly tightened from (0.99, 0.43) to (0.85, 0.36).