

# Financial Fragility with SAM?

Daniel L. Greenwald, Tim Landvoigt, Stijn Van Nieuwerburgh\*

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## Abstract

Shared Appreciation Mortgages feature mortgage payments that adjust with house prices. They are designed to stave off borrower default by providing payment relief when house prices fall. Some argue that SAMs may help prevent the next foreclosure crisis. However, the home owners gains from payment relief are the mortgage lenders losses. A general equilibrium model where financial intermediaries channel savings from saver to borrower households shows that indexation of mortgage payments to aggregate house prices increases financial fragility, reduces risk-sharing, and leads to expensive financial sector bailouts. In contrast, indexation to local house prices reduces financial fragility and improves risk-sharing.

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\*First draft: November 6, 2017. Greenwald: Massachusetts Institute of Technology Sloan School; email: [dlg@mit.edu](mailto:dlg@mit.edu). Landvoigt: University of Pennsylvania Wharton School, NBER, and CEPR; email: [tim-land@wharton.upenn.edu](mailto:tim-land@wharton.upenn.edu). Van Nieuwerburgh: Columbia University Graduate School of Business, NBER, and CEPR, 3022 Broadway, Uris Hall 809, New York, NY 10027; email: [svnieuw@gsb.columbia.edu](mailto:svnieuw@gsb.columbia.edu). We are grateful for comments from Adam Guren and Erik Hurst, from conference discussants Nina Boyarchenko, Zhiguo He, Yunzhi Hu, Tim McQuade, Kurt Mitman, Fang Yang, and Jiro Yoshida, and from conference and seminar participants at the SED in Edinburgh, Philadelphia Fed, St. Louis Fed, Columbia GSB, Princeton, HEC Montreal, Wharton, the Bank of Canada Annual Conference, the FRB Atlanta/GSU Real Estate Conference, the UNC Junior Roundtable, NYU Stern, HULM, University of Melbourne, UNSW in Sydney, University of Colorado at Boulder, MIT Sloan finance, the NYC real estate conference at Baruch, MIT economics, the AREUEA National Conference, the BI-SHoF conference in Stockholm, the pre-WFA real estate conference in San Diego, the NBER SI Real Estate meeting in Cambridge, the CEPR conference in Gerzensee, and the EFA meeting in Warsaw.

# 1 Introduction

The \$10 trillion market in U.S. mortgage debt is the world's largest consumer debt market and its second largest fixed income market. Mortgages are not only the largest liability for U.S. households, they are also the largest asset of the U.S. financial sector.<sup>1</sup> Given the heavy exposure of the financial sector to mortgages, large house price declines and the default waves that accompany them can severely hurt the solvency of the U.S. financial system. This became painfully clear during the Great Financial Crisis of 2008-2011, as U.S. house prices fell by 30% nationwide, and by much more in some regions, pushing roughly 25% of U.S. home owners underwater by 2010, and leading to seven million foreclosures. Large losses on real estate loans caused several U.S. banks to collapse during the crisis, while the stress to surviving banks' balance sheets led them to dramatically tighten mortgage lending standards, precluding many home owners from refinancing into lower interest rates.<sup>2</sup> Homeowners' reduced ability to tap into their housing wealth short-circuited the stimulative consumption response from lower mortgage rates that policy makers had hoped for.

This experience led economists and policy makers to ask whether a different mortgage finance system would result in a better risk sharing arrangement between borrowers and lenders.<sup>3</sup> While contracts offering alternative allocations of interest rate risk are already widely available — most notably, the adjustable rate mortgage (ARM), which offers nearly perfect pass-through of interest rates — contracts offering alternative divisions of *house price* risk are still rare. Recently, however, some fintech lenders have begun to offer such contracts — most notably the shared appreciation mortgage (SAM), which indexes mortgage payments to house price changes.<sup>4</sup>

A SAM contract ensures that the borrower receives payment relief in bad states of the world, potentially reducing mortgage defaults and the associated deadweight losses to

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<sup>1</sup>Banks and credit unions hold \$3 trillion in mortgage loans directly on their balance sheets in the form of whole loans, and an additional \$2.2 trillion in the form of mortgage-backed securities. Including insurance companies, money market mutual funds, broker-dealers, and mortgage REITs in the definition of the financial sector adds another \$1.5 trillion to the financial sector's agency MBS holdings. Adding the Federal Reserve Bank and the GSE portfolios adds a further \$2 trillion and increases the share of the financial sector's holdings of agency MBS to nearly 80%.

<sup>2</sup>Charge-off rates of residential real estate loans at U.S. banks went from 0.1% in mid-2006 to 2.8% in mid-2009, returning to their initial value only in mid-2016.

<sup>3</sup>The New York Federal Reserve Bank organized a two-day conference on this topic in May 2015.

<sup>4</sup>Examples of startups in this space are Unison Home Ownership Investors, Point Digital Finance, Own Home Finance, and Patch Homes. In addition, similar contracts have been offered to faculty at Stanford University for leasehold purchases over the past fifteen years (Landvoigt, Piazzesi, and Schneider, 2014).

society. However, SAMs impose losses on mortgage lenders in these adverse aggregate states, which may increase financial fragility at inopportune times. Our paper is the first to study how SAM contracts affect the allocation of house price risk between mortgage borrowers, financial intermediaries, and savers in a general equilibrium framework. It proposes a shift in the mortgage design literature from a focus on *household risk management* to one on *system-wide risk management*. The main goal of this paper is to quantitatively assess whether SAMs present a better arrangement to the overall economy than standard fixed-rate mortgages (FRMs).

We begin with a rich baseline model where mortgage borrowers obtain long-term, defaultable, prepayable, nominal mortgages from financial intermediaries. These intermediaries are financed with short-term deposits raised from savers and equity raised from their shareholders, subject to realistic capital requirements, and are bailed out by the government in case of insolvency. Borrowers face idiosyncratic house valuation shocks while banks face idiosyncratic profit shocks, which influence their respective optimal default decisions. We solve the model using a state-of-the-art global non-linear solution technique that allows for occasionally binding constraints.

To evaluate the mortgage system's resilience to adverse scenarios, our model economy transits between a normal state and a crisis state featuring high house price uncertainty and a fall in aggregate home values, in addition to aggregate business-cycle income risk. Under standard FRMs, the arrival of a crisis state leads to higher rates of borrower defaults, bank losses, and bank failures, along with large falls in borrower consumption as the financial sector contracts.

To study the impact of alternative mortgage contracts, we consider SAM economies where mortgage payments are either indexed to aggregate house prices or to local house prices. We contrast the effects of alternative schemes on the model's key externalities: the deadweight losses and risk-sharing consequences of borrower and bank default. Our main result is that indexation to aggregate (national) house prices reduces borrower welfare even though it slightly reduces mortgage defaults, due to a severe increase in financial fragility. These contracts lead mortgage lenders to absorb aggregate house price declines, causing a wave of bank failures and triggering bailouts ultimately funded by taxpayers, including the borrowers. Equilibrium house prices are lower and fall more in crises with aggregate indexation. Ironically, intermediary welfare increases as they enjoy large gains from increased mortgage payments in housing expansions, and can charge higher mortgage spreads in a riskier financial system.

In sharp contrast, indexation of mortgage payments to the local component of house price risk only can eliminate up to half of mortgage defaults while reducing systemic risk. Banks' geographically diversified portfolios of SAMs allow them to offset the cost of debt forgiveness in areas where house prices fall by collecting higher mortgage payments from areas where house prices rise. Lower mortgage defaults in turn substantially reduce bank failures and dampen fluctuations in intermediary net worth, stabilizing the financial system, and reducing deadweight losses. Banking becomes safer, but also less profitable, due to a fall in mortgage spreads and in the value of the bailout option. As a result, welfare of borrowers and savers rises, at the expense of bank owners. A combination of aggregate and local indexation, which we label regional indexation, generates modest welfare benefits to the economy.

Applying these insights, we examine the consequences of several realistic SAM implementations. Indexing interest payments only — which are fixed only until the next borrower mortgage transaction — has much weaker effects than indexing principal. Asymmetric indexation, which allows payments to fall but never to rise, dramatically decreases default rates, but does so by shrinking average household leverage, rather than by improving risk sharing. Our results imply that macrofinancial considerations should play an important role in the design of such contracts. We close with a series of robustness checks showing that our results continue to hold when bank bailouts are financed with government debt rather than instantaneous taxation, and when mortgage defaults have both a strategic and a liquidity component.

**Literature Review.** This paper contributes to the literature that studies innovative mortgage contracts, such as [Shiller and Weiss \(1999\)](#), who discuss the idea of home equity insurance policies. SAMs were first discussed in detail in a series of papers by [Caplin, Chan, Freeman, and Tracy \(1997\)](#); [Caplin, Carr, Pollock, and Tong \(2007\)](#); [Caplin, Cunningham, Engler, and Pollock \(2008\)](#). They envision a SAM as a second mortgage in addition to a conventional FRM with a smaller principal balance.<sup>5</sup> They emphasize that SAMs are not only a valuable work-out tool after a default has taken place, but are also useful to prevent a mortgage crisis in the first place. More recently [Mian and Sufi \(2014\)](#) have proposed a Shared Responsibility Mortgage (SRM), a first mortgage whose payments fall

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<sup>5</sup>This SAM has no interest payments and its principal needs to be repaid upon termination (e.g., sale of the house). At that point the borrower shares a fraction of the house value appreciation with the lender, but only if the house has appreciated in value. The result is lower monthly mortgage payments throughout the life of the loan, which can enhance affordability and improve sharing of housing risk.

when the local house price index goes down, and return to the initial payment upon recovery, while lenders receive a share of home value appreciation upon sale. They argue that foreclosure avoidance raises house prices in a SRM world and shares wealth losses more equitably between borrowers and lenders, boosting borrower spending and aggregate consumption after house price falls. We build on this literature through our analysis of intermediary and financial risk, which interacts with the borrower balance sheet risk discussed in these works.

[Kung \(2015\)](#) studies the effect of the disappearance of non-agency mortgages for house prices, mortgage rates and default rates in an industrial organization model of the Los Angeles housing market. While not the emphasis of his work, he also evaluates the hypothetical introduction of SAMs in the 2003-07 period, finding that SAMs would have enjoyed substantial uptake, partially supplanting non-agency loans. However, SAMs would have further exacerbated the boom and would not have mitigated the bust. Our work complements this approach by providing an equilibrium model of the entire U.S. housing market, with risk averse lenders, and endogenously determined risk-free rate and mortgage risk premium. This framework captures important effects as banks owned by risk averse shareholders are negatively affected by aggregate house price declines, allowing mortgage payment indexation to potentially exacerbate financial fragility.

[Piskorski and Tchisty \(2018\)](#) also study mortgage design in a stylized, risk neutral environment. They emphasize asymmetric information about home values between borrowers and lenders and derive the optimal mortgage contract. The latter takes the form of a Home Equity Insurance Mortgage that eliminates the strategic default option and insures borrower's home equity. Our emphasis on imperfect risk sharing and financial fragility complements their approach.

[Guren, Krishnamurthy, and McQuade \(2018\)](#) and [Campbell, Clara, and Cocco \(2018\)](#) investigate the interaction of ARM and FRM contracts with monetary policy. They study an FRM that costlessly converts to an ARM in a crisis so as to provide concentrated payment relief in a crisis. The former paper solves for house prices but has risk neutral lenders, while the latter paper introduces risk averse lenders but takes house prices and interest rates as given. These authors focus on interest rate risk, contrasting e.g., adjustable-rate and fixed-rate mortgages.<sup>6</sup> Since interest rate risk is easier for banks

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<sup>6</sup>Related work on contract schemes other than house price indexation include [Piskorski and Tchisty \(2011\)](#), who study optimal mortgage contract design in a partial equilibrium model with stochastic house prices and show that option-ARM implements the optimal contract; [Kalotay, 2015](#)), who considers automatically refinancing mortgages or ratchet mortgages (whose interest rate only adjusts down); and [Eberly and Krishnamurthy \(2014\)](#), who propose a mortgage contract that automatically refinances from a FRM

to hedge than house price risk, these authors abstract from implications for financial fragility, instead emphasizing a rich borrower risk profile that includes a life cycle and uninsurable idiosyncratic income risk. In contrast, our framework considers house price risk that is difficult for banks to hedge, and emphasizes of the intermediation sector. We see both of these approaches as highly complementary to our own.

More generally, our paper connects to the quantitative macro-housing literature, providing a novel and tractable general equilibrium setting for analyzing the interaction between the housing and financial sectors.<sup>7</sup> Our paper also contributes to the literature that studies the amplification of business cycle shocks provided by credit frictions, focusing specifically on key features of the mortgage market.<sup>8</sup> Finally, we provide a general equilibrium counterpart to recent empirical work that has found strong responses of consumption and default rates to changes in mortgage interest rates and house prices.<sup>9</sup>

**Overview.** The rest of the paper proceeds as follows. Section 2 presents the theoretical model, while Section 3 discusses its calibration. The main results are in Section 4, with extensions presented in Section 5. Section 6 concludes. Model derivations, first order conditions characterizing the solution, and additional results are relegated to the appendix.

## 2 Model

### 2.1 Demographics

The economy is populated by a continuum of agents of three types: borrowers (denoted  $B$ ), depositors (denoted  $D$ ), and intermediaries (denoted  $I$ ). The measure of type  $j$  in the

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into an ARM, even when the loan is underwater.

<sup>7</sup> [Elenev, Landvoigt, and Van Nieuwerburgh \(2016\)](#) studies the role the default insurance provided by the government-sponsored enterprises. [Gete and Zecchetto \(2018\)](#) studies the redistributive role of the Federal Housing Agency. [Greenwald \(2018\)](#) studies the interaction between payment-to-income and loan-to-value constraints in a model of monetary shock transmission through the mortgage market, but without default. [Favilukis, Ludvigson, and Van Nieuwerburgh \(2017\)](#) study the role of relaxed down payment constraints in explaining the house price boom. [Corbae and Quintin \(2014\)](#) investigate the effect of risky mortgage innovation in a general equilibrium model with default. [Guren and McQuade \(2017\)](#) study the interaction of foreclosures and house prices in a model with search.

<sup>8</sup>See, e.g., [Bernanke and Gertler \(1989\)](#), [Bernanke, Gertler, and Gilchrist \(1996\)](#), [Kiyotaki and Moore \(1997\)](#), and [Gertler and Karadi \(2011\)](#). A second generation of models has added nonlinear dynamics and a richer financial sector. E.g., [Brunnermeier and Sannikov \(2014\)](#), [He and Krishnamurthy \(2012\)](#), [He and Krishnamurthy \(2013\)](#), [He and Krishnamurthy \(2014\)](#), [Gârleanu and Pedersen \(2011\)](#), [Adrian and Boyarchenko \(2012\)](#), [Maggiori \(2013\)](#), [Moreira and Savov \(2016\)](#), and [Elenev, Landvoigt, and Van Nieuwerburgh \(2017\)](#).

<sup>9</sup>See e.g., [Mian and Sufi \(2009\)](#); [Mian, Rao, and Sufi \(2013\)](#), [Di Maggio, Kermani, Keys, Piskorski, Ramcharan, Seru, and Yao \(2017\)](#), [Fuster and Willen \(2015\)](#).

population is denoted  $\chi_j$ , with  $\chi_B + \chi_D + \chi_I = 1$ .

## 2.2 Endowments

The two consumption goods in the economy — nondurable consumption and housing services — are provided by two Lucas trees. The overall endowment  $Y_t$  is equal to a stationary component  $\tilde{Y}_t$  scaled by a deterministic component that grows at a constant rate  $g$ :

$$Y_t = e^{gt} \tilde{Y}_t,$$

where  $\mathbb{E}(\tilde{Y}_t) = 1$  and

$$\log \tilde{Y}_t = (1 - \rho_y) \mu_y + \rho_y \log \tilde{Y}_{t-1} + \sigma_y \varepsilon_{y,t}, \quad \varepsilon_{y,t} \sim N(0, 1). \quad (1)$$

The  $\varepsilon_{y,t}$  represent transitory shocks to the level of aggregate labor income. For nondurable consumption, each agent type  $j$  receives a fixed share  $s_j$  of the overall endowment  $Y_t$ , which cannot be traded.

Shares of the housing tree are in fixed total supply  $\bar{K}$ , produce housing services proportional to the stock, and grow at the same rate  $g$  as the nondurable endowment. Household-owned housing requires a maintenance cost of fraction  $\nu^K$  of its value per period. To ensure that a borrower is the marginal pricer of housing, we fix intermediary and depositor demand for housing to be  $H_t^I = \bar{K}^I$  and  $H_t^D = \bar{K}^D$ .

## 2.3 Preferences

To allow for non-trivial risk premia, we assume that an agent of type  $j \in \{B, D, I\}$  has preferences following [Epstein and Zin \(1989\)](#), so that lifetime utility is given by

$$U_t^j = \left\{ (1 - \beta_j) (u_t^j)^{1-1/\psi} + \beta_j \left( \mathbb{E}_t \left[ (U_{t+1}^j)^{1-\gamma_j} \right] \right)^{\frac{1-1/\psi}{1-\gamma_j}} \right\}^{\frac{1}{1-1/\psi}} \quad (2)$$

$$u_t^j = (C_t^j)^{1-\zeta_t} (H_t^j)^{\zeta_t} \quad (3)$$

where  $C_t^j$  is nondurable consumption and  $H_t^j$  is housing services, and the preference parameter  $\zeta_t$  is allowed to vary with the state of the economy. Housing capital produces housing services with a linear technology. We denote by  $\Lambda^j$  the intratemporal marginal rate of substitution (or stochastic discount factor) of agent  $j$ .

## 2.4 Financial Technology

To allow for aggregation, we assume that households are able to trade a complete set of state-dependent securities with households of their own type, providing perfect insurance against idiosyncratic consumption risk, but cannot trade these securities with members of the other types. Between-type trade is limited to two financial assets: mortgages that can be traded between the borrower and the intermediary, and deposits that can be traded between the depositor and the intermediary.<sup>10</sup>

**Mortgage Contracts.** Mortgage contracts are modeled as nominal perpetuities with payments that decline geometrically, so that one unit of debt yields the payment stream  $1, \delta, \delta^2, \dots$  until prepayment or default. The interest portion of mortgage payments can be deducted from taxes. New mortgages face a loan-to-value constraint (shown below in (11)) that is applied at origination only, meaning that borrowers do not have to delever if they violate the constraint later on.

**Borrower Refinancing.** Non-defaulting borrowers can choose at any time to obtain a new mortgage loan and simultaneously re-optimize their housing position. If a refinancing borrower previously held a mortgage, she must first prepay the principal balance on the existing loan before taking on a new loan. Since borrowers in the model borrow up to their credit limits when taking out new loans — as is typical in reality — adjustments in borrower leverage largely occur through the frequency at which new loans are issued. Since leverage is a key state variable for default, this realistic model of mortgage refinancing allows us to capture a potentially important channel influencing financial fragility.

Following [Greenwald \(2018\)](#), the transaction cost of obtaining a new loan is proportional to the balance on the new loan  $M_t^*$ , defined as  $\kappa_{i,t}M_t^*$ , where  $\kappa_{i,t}$  is drawn i.i.d. across borrowers and time from a distribution with CDF  $\Gamma_\kappa$ . Since these costs largely stand in for non-monetary frictions such as inertia, they are rebated to borrowers and do not impose an aggregate resource cost. We assume that borrowers must commit in advance to a refinancing policy that can depend in an unrestricted way on  $\kappa_{i,t}$  and all current values and expectations of aggregate variables, but cannot depend on the borrower's individual loan characteristics. This setup keeps the problem tractable by removing the distribution

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<sup>10</sup>Equivalently, households are able to trade a complete set of state-dependent securities with households of their own type, providing perfect insurance against idiosyncratic consumption risk, but cannot trade these securities with members of the other types. Hence, our model features incomplete risk sharing which can potentially be improved by mortgage indexation.



of loans as a state variable while maintaining the realistic feature that an endogenous fraction of borrowers choose to refinance in each period and that this fraction responds endogenously to the state of the economy.

We guess and verify that the optimal plan for the borrower is to refinance whenever  $\kappa_{i,t} \leq \bar{\kappa}_t$ , where  $\bar{\kappa}_t$  is a threshold cost that makes the borrower indifferent between refinancing and not refinancing. The fraction of non-defaulting borrowers who choose to refinance is therefore

$$Z_{R,t} = \Gamma_\kappa(\bar{\kappa}_t).$$

Once the threshold cost (equivalently, refinancing rate) is known, the total transaction cost per unit of debt is defined by

$$\Psi_t(Z_{R,t}) = \int^{\bar{\kappa}_t} \kappa d\Gamma_\kappa = \int^{\Gamma_\kappa^{-1}(Z_{R,t})} \kappa d\Gamma_\kappa.$$

**Borrower Default and Mortgage Indexation.** Before deciding whether to refinance a loan, borrowers can choose to default on the loan. Upon default, the housing collateral backing the loan is seized by the intermediary. To allow an aggregated model in which the default rate responds endogenously to macroeconomic conditions, we introduce stochastic processes  $\omega_{i,t}$  for each borrower  $i$  that influence the quality of borrowers' houses.

In practice, SAM contracts typically propose indexing to a local house price index rather than to individual house values to avoid moral hazard issues relating to the maintenance of the property. To accommodate this, we decompose house quality into two components,  $\omega_{i,t} = \omega_{i,t}^L \omega_{i,t}^U$ , where  $\omega_{i,t}^L$  is local component that shifts prices in an area relative to the national average — and can potentially be insured by mortgage contracts — while  $\omega_{i,t}^U$  is an uninsurable component that shifts an individual house price relative to its local area. These components are drawn i.i.d. from independent log-normal distributions

$$\log \omega_{i,t}^L \sim N\left(-\frac{1}{2}\alpha\sigma_{\omega,t}^2, \alpha\sigma_{\omega,t}^2\right) \quad (4)$$

$$\log \omega_{i,t}^U \sim N\left(-\frac{1}{2}(1-\alpha)\sigma_{\omega,t}^2, (1-\alpha)\sigma_{\omega,t}^2\right) \quad (5)$$

ensuring that each process has mean unity, and that the local and uninsurable components account for  $\alpha$  and  $1 - \alpha$  of the cross-sectional variance of  $\omega_{i,t}$ , respectively. The overall dispersion  $\sigma_{\omega,t}$  is allowed to vary between normal times and financial recessions.<sup>11</sup>

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<sup>11</sup>Local and individual house values in reality are persistent rather than i.i.d. However, for the case of

In addition to the standard mortgage contracts defined above, we introduce Shared Appreciation Mortgages whose payments are indexed to house prices. We allow SAM contracts to insure households in two ways. First, mortgage payments can be indexed to the aggregate house price  $p_t$ . In this case, the principal balance and payment on each existing mortgage loan are multiplied each period by:

$$\zeta_{p,t} = \left( \frac{p_t}{p_{t-1}} \right)^{\iota_p}. \quad (6)$$

The special cases  $\iota_p = 0$  and  $\iota_p = 1$  correspond to the cases of no insurance and complete insurance against aggregate house price risk.

Second, mortgage contracts can be indexed against shocks to the individual house qualities  $\omega_{i,t}$ . We assume that the uninsurable component  $\omega_{i,t}^U$  cannot be indexed due to moral hazard risk, but that the local component  $\omega_{i,t}^L$  can be insured. Specifically, each period, the principal balance and interest payment on the loan backed by a house that experiences regional house quality growth  $\omega_{i,t}^L$  are multiplied by:

$$\zeta_{\omega}(\omega_{i,t}^L) = \left( \omega_{i,t}^L \right)^{\iota_{\omega}}. \quad (7)$$

The special cases  $\iota_{\omega} = 0$  and  $\iota_{\omega} = 1$  correspond to zero insurance and complete insurance against cross-sectional local house price risk, respectively.

As with refinancing, borrowers must commit to a default plan that can depend in an unrestricted way on  $\omega_{i,t}^L, \omega_{i,t}^U$ , and the aggregate states, but not on a borrower's individual loan conditions. We guess and verify that the optimal plan for the borrower is to default whenever  $\omega_{i,t}^U \leq \bar{\omega}_t^U$ , where  $\bar{\omega}_t^U$  is the threshold value of uninsurable (individual-level) house quality that makes a borrower indifferent between defaulting and not defaulting. The level of the default threshold depends on the aggregate state, the insurable local component  $\omega_{i,t}^L$ , also on the level of mortgage payment indexation. Given  $\bar{\omega}_t^U$ , the fraction of non-defaulting borrowers is

$$Z_{N,t} = \int \left( 1 - \Gamma_{\omega,t}^U(\bar{\omega}_t^U) \right) d\Gamma_{\omega,t}^L$$

where  $\Gamma_{\omega,t}^U$  and  $\Gamma_{\omega,t}^L$  are the CDFs of  $\omega_{i,t}^U$  and  $\omega_{i,t}^L$ , respectively, and where the integral

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symmetric indexation, the i.i.d. specification delivers identical results to more general AR(1) processes (given our other modeling assumptions on risk sharing within the borrower collective). Discussion and details of the equivalent AR(1) formulation can be found in appendix C.2.

is needed because  $\bar{\omega}_t^U$  depends on  $\omega_{i,t}^L$ . The share of housing kept by non-defaulting borrower households is

$$Z_{K,t} = \int \left( \int_{\omega_{i,t}^U > \bar{\omega}_t^U} \omega_{i,t}^U d\Gamma_{\omega,t}^U \right) \omega_{i,t}^L d\Gamma_{\omega,t}^L. \quad (8)$$

where inner-most integral contains this selection effect — borrowers only keep their housing when their idiosyncratic quality shock was sufficiently good — while the outer integral again accounts for dependence of  $\bar{\omega}_t^U$  on local house quality.

The fractions of principal and interest payments retained by the borrowers are defined by  $Z_{M,t}$  and  $Z_{A,t}$ , respectively, and are given by

$$Z_{M,t} = Z_{A,t} = \int \int \underbrace{\left( 1 - \Gamma_{\omega,t}^U \left( \bar{\omega}_t^U \right) \right)}_{\text{remove defaulters}} \underbrace{\left( \omega_{i,t}^L \right)^{\iota_\omega}}_{\text{indexation}} d\Gamma_{\omega,t}^L. \quad (9)$$

The first term in the integral above removes the fraction of debt that is defaulted on and is not repaid, while the second component adjusts for indexation of debt to local prices.<sup>12</sup>

It is straightforward to show that for the limiting case when all cross-sectional house price risk is insurable ( $\alpha = 1$ ) and this risk is fully indexed ( $\iota_\omega = 1$ ), we obtain  $Z_{N,t} = Z_{M,t} = Z_{A,t} = Z_{K,t} = 1$ , in which case borrowers' optimal policy is to never default on any payments. In contrast, under a standard mortgage contract with no indexation ( $\iota_p = \iota_\omega = 0$ ), we have  $Z_{M,t} = Z_{A,t} = Z_{N,t}$ , so that conditional on non-default, neither debt balances nor interest payments are directly influenced by local house prices.

**REO Sector.** The housing collateral backing defaulted loans is seized by the intermediary and rented out as REO (“real estate owned”) housing to the borrower. Housing in this state incurs a larger maintenance cost than usual,  $\nu^{REO} > \nu^K$ , designed to capture losses from foreclosure. With probability  $S^{REO}$  per period, REO housing is sold back to borrowers as owner-occupied housing. The existing stock of REO housing is denoted by  $K_t^{REO}$ , and the value of a unit of REO-owned housing is denoted  $p_t^{REO}$ .

**Deposit Technology.** Deposits in the model take the form of risk-free one-period loans issued from the depositor to the intermediary, where the price of these loans is denoted  $q_t^f$ , implying the interest rate  $1/q_t^f$ . Intermediaries must satisfy a leverage constraint (defined

<sup>12</sup>While  $Z_{A,t}$  and  $Z_{M,t}$  are identical in this baseline case, it is convenient to define them separately since they will diverge under separate indexation of interest and principal in Section 5.

below in (21)) stating that their promised deposit repayments must be collateralized by their existing loan portfolio.

## 2.5 Borrower's Problem

Given this model setup, the individual borrower's problem aggregates to that of a representative borrower. The endogenous state variables are the promised payment  $A_t^B$ , the face value of principal  $M_t^B$ , and the stock of borrower-owned housing  $K_t^B$ . The representative borrower's control variables are nondurable consumption  $C_t^B$ , housing service consumption  $H_t^B$ , the amount of housing  $K_t^*$  and new loans  $M_t^*$  taken on by refinancers, the refinancing fraction  $Z_{R,t}$ , and the default policy  $\bar{\omega}_t^U$ , which implicitly determines  $(Z_{N,t}, Z_{M,t}, Z_{A,t}, Z_{K,t})$ .

The borrower maximizes (2) subject to the budget constraint:

$$\begin{aligned}
C_t^B = & \underbrace{(1 - \tau)Y_t^B}_{\text{disp. income}} + \underbrace{Z_{R,t} \left( Z_{N,t}M_t^* - \delta Z_{M,t}M_t^B \right)}_{\text{net new borrowing}} - \underbrace{(1 - \delta)Z_{M,t}M_t^B}_{\text{principal payment}} - \underbrace{(1 - \tau)Z_{A,t}A_t^B}_{\text{interest payment}} \\
& - \underbrace{p_t \left[ Z_{R,t}Z_{N,t}K_t^* + (v^K - Z_{R,t})Z_{K,t}K_t^B \right]}_{\text{owned housing}} - \underbrace{\rho_t \left( H_t^B - K_t^B \right)}_{\text{rental housing}} \\
& - \underbrace{(\Psi(Z_{R,t}) - \bar{\Psi}_t)Z_{N,t}M_t^*}_{\text{net transaction costs}} - \underbrace{T_t^B}_{\text{lump sum taxes}}
\end{aligned} \tag{10}$$

the loan-to-value constraint

$$M_t^* \leq \phi^K p_t K_t^* \tag{11}$$

and the laws of motion

$$M_{t+1}^B = \bar{\pi}^{-1} \zeta_{p,t+1} \left[ Z_{R,t} Z_{N,t} M_t^* + \delta (1 - Z_{R,t}) Z_{M,t} M_t^B \right] \tag{12}$$

$$A_{t+1}^B = \bar{\pi}^{-1} \zeta_{p,t+1} \left[ Z_{R,t} Z_{N,t} r_t^* M_t^* + \delta (1 - Z_{R,t}) Z_{A,t} A_t^B \right] \tag{13}$$

$$K_{t+1}^B = Z_{R,t} Z_{N,t} K_t^* + (1 - Z_{R,t}) Z_{K,t} K_t^B \tag{14}$$

where  $\bar{\pi}$  is the inflation rate (assumed constant),  $r_t^*$  is the interest rate on new mortgages,  $\tau$  is the income tax rate, which also applies to the mortgage interest deductibility,  $\rho_t$  is the rental rate for housing services,  $\bar{\Psi}_t$  is a subsidy that rebates transaction costs back to borrowers, and  $T_t^B$  are taxes raised on borrowers to pay for intermediary bailouts (defined below in (29)). Aggregate indexation influences the problem by directly scaling debt and

interest payments ( $M_{t+1}^B$  and  $A_{t+1}^B$ ) to aggregate house price growth, while local indexation (whose direct effects wash out in aggregate) instead influences the default decision ( $Z_{N,t}, Z_{M,t}, Z_{A,t}, Z_{K,t}$ ).

## 2.6 Intermediary's Problem

The intermediation sector consists of intermediary households, mortgage lenders (banks), and REO firms. The intermediary households, who we will refer to as “bank owners,” are equity holders of both the banks and the REO firms. Each period, the bank owners receive income  $Y_t^I$ , and the aggregate dividends  $D_t^I$  and  $D_t^{REO}$  from banks and REO firms, respectively (defined in equations (27) and (30) below). Bank owners choose consumption  $C_t^I$  to maximize (2) subject to the budget constraint:

$$C_t^I \leq (1 - \tau)Y_t^I + D_t^I + D_t^{REO} - v^K p_t H_t^I - T_t^I, \quad (15)$$

where  $T_t^I$  are taxes raised on intermediary households to pay for bank bailouts (defined in (29) below). Intermediary households consume their fixed endowment of housing services each period,  $H_t^I = \bar{K}^I$ .

Banks and REO firms maximize shareholder value. Banks lend to borrowers, issue deposits, and trade in the secondary market for mortgage debt. They are subject to idiosyncratic profit shocks and have limited liability, i.e., they optimally decide whether to default at the beginning of each period. When a bank defaults, it is seized by the government, which guarantees its deposits. The equity of the defaulting bank is wiped out, and bank owners set up a new bank in place of the bankrupt one.

REO firms buy foreclosed houses from banks, rent these REO houses to borrowers, and sell REO housing in the regular housing market after maintenance.

**Bank Portfolio Choice.** Each bank chooses a portfolio of mortgage loans and how many deposits to issue. Although each mortgage with a different interest rate has a different secondary market price, we show in the appendix that any portfolio of loans can be replicated using only two instruments: an interest-only (IO) strip, and a principal-only (PO) strip. Let  $A_t^I$  and  $M_t^I$  denote start-of-period holdings of IO and PO strips, respectively, which correspond to total promised interest payments and principal balances ( $A_t^B$  and  $A_t^I$ ) at equilibrium. If we denote new lending by  $L_t^*$ , then the *supply* of IO and PO strips

available on the secondary market is given by

$$\hat{M}_t^I = L_t^* + \delta(1 - Z_{R,t})Z_{M,t}M_t^I \quad (16)$$

$$\hat{A}_t^I = r_t^*L_t^* + \delta(1 - Z_{R,t})Z_{A,t}A_t^I. \quad (17)$$

Next, denote bank *demand* for PO and IO strips (desired end-of-period holdings), by  $\tilde{M}_t^I$  and  $\tilde{A}_t^I$ , respectively. In equilibrium, market clearing implies  $\hat{M}_t^I = \tilde{M}_t^I$  and  $\hat{A}_t^I = \tilde{A}_t^I$ . The laws of motion start-of-period IO and PO strip holdings are therefore

$$M_{t+1}^I = \bar{\pi}^{-1}\zeta_{p,t+1}\tilde{M}_t^I \quad (18)$$

$$A_{t+1}^I = \bar{\pi}^{-1}\zeta_{p,t+1}\tilde{A}_t^I. \quad (19)$$

which depend on both inflation (since the contracts are nominal) and indexation. The market value of the portfolio held by banks at the end of each period is

$$J_t^I = \underbrace{(1 - r_t^*q_t^A - q_t^M)L_t^*}_{\text{net new debt}} + \underbrace{q_t^A\tilde{A}_t^I}_{\text{IO strips}} + \underbrace{q_t^M\tilde{M}_t^I}_{\text{PO strips}} - \underbrace{q_t^f B_{t+1}^I}_{\text{new deposits}} \quad (20)$$

where  $q_t^A$  and  $q_t^M$  are the market prices of IO and PO strips, respectively. This portfolio is chosen by banks subject to a leverage constraint

$$B_{t+1}^I \leq \phi^I \left( q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I \right) \quad (21)$$

that limits the amount of deposit finance to a fraction of their assets. Since banks enjoy limited liability and can issue insured deposits, they have incentives to take on excessive risk in the form of high leverage. The constraint represents a regulatory equity capital requirement that limits bank risk taking.

To calculate the payoff of this portfolio in period  $t + 1$ , we first define the recovery rate of housing from foreclosed borrowers, per unit of face value outstanding, as<sup>13</sup>

$$X_t = \frac{(1 - Z_{K,t})K_t^B(p_t^{REO} - v^{REO}p_t)}{M_t^B}. \quad (22)$$

After paying maintenance on the REO housing for one period, the banks sell the seized houses to the REO sector at prices  $p^{REO}$ .

<sup>13</sup>Note that  $X_t$  is taken as given by each individual bank. A bank does not internalize the effect of its mortgage debt issuance on the overall recovery rate.

Combining the above, a bank's portfolio payoff is:

$$W_{t+1}^I = \underbrace{\left[ X_{t+1} + Z_{M,t+1} \left( (1 - \delta) + \delta Z_{R,t+1} \right) \right] M_{t+1}^I + Z_{A,t+1} A_{t+1}^I}_{\text{payments on existing debt}} + \underbrace{\delta(1 - Z_{R,t+1}) \left( Z_{A,t+1} q_{t+1}^A A_{t+1}^I + Z_{M,t+1} q_{t+1}^M M_{t+1}^I \right)}_{\text{sales of IO and PO strips}} - \underbrace{\bar{\pi}^{-1} B_t^I}_{\text{deposit redemptions}} \quad (23)$$

which is net worth of banks at the beginning of period  $t + 1$ .

**Bank's Problem.** Denote by  $\mathcal{S}_t^I$  all state variables exogenous to banks. At the beginning of each period, before making their optimal default decision, banks receive an idiosyncratic profit shock  $\epsilon_t^I \sim F_\epsilon^I$ , with  $E(\epsilon_t^I) = 0$ . The value of banks that do not default can be expressed recursively as:

$$V_{ND}^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I - \epsilon_t^I + E_t \left[ \Lambda_{t,t+1}^I \max \left\{ V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I), 0 \right\} \right], \quad (24)$$

subject to the bank leverage constraint (21), the definitions of  $J_t^I$  and  $W_t^I$  in (20) and (23), respectively, and the transition laws for the aggregate supply of IO and PO strips in (16) – (19). The value of defaulting banks to shareholders is zero. The value of the newly started bank that replaces a bank liquidated by the government after defaulting, is given by:

$$V_R^I(\mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} - J_t^I + E_t \left[ \Lambda_{t,t+1}^I \max \left\{ V_{ND}^I(W_{t+1}^I, \mathcal{S}_{t+1}^I), 0 \right\} \right], \quad (25)$$

subject to the same set of constraints as the non-defaulting bank.

Beginning-of-period net worth  $W_t^I$  and the idiosyncratic profit shock  $\epsilon_t^I$  are irrelevant for the portfolio choice of newly started banks. Inspecting equation (24), one can see that the optimization problem of non-defaulting banks is also independent of  $W_t^I$  and  $\epsilon_t^I$ , since the value function is linear in those variables and they are determined before the portfolio decision. Taken together, this implies that all banks will choose identical portfolios at the end of the period. In the appendix, we show that we can define a value function after the default decision to characterize the portfolio problem of all banks:<sup>14</sup>

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + E_t \left[ \Lambda_{t,t+1}^I F_{\epsilon,t+1}^I \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right], \quad (26)$$

<sup>14</sup>The value of the newly started bank with zero net worth is simply the value in (26) evaluated at  $W_t^I = 0$ .

where

$$F_{\epsilon,t+1}^I \equiv F_{\epsilon}^I(V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I))$$

is the probability of continuation, and  $\epsilon_{t+1}^{I,-} = E[\epsilon_{t+1}^I | \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I)]$  is the expectation of  $\epsilon_{t+1}^I$  conditional on continuation. The objective in (26) is subject to the same set of constraints as (24).

**Aggregation and Government Deposit Guarantee.** By the law of large numbers, the fraction of defaulting banks each period is  $1 - F_{\epsilon,t}^I$ . The aggregate dividend paid by banks to their shareholders, the intermediary households, is:

$$\begin{aligned} D_t^I &= F_{\epsilon,t}^I (W_t^I - \epsilon_t^{I,-} - J_t^I) - (1 - F_{\epsilon,t}^I) J_t^I \\ &= F_{\epsilon,t}^I (W_t^I - \epsilon_t^{I,-}) - J_t^I. \end{aligned} \quad (27)$$

Bank shareholders bear the burden of replacing liquidated banks by an equal measure of new banks and seeding them with new capital equal to that of continuing banks ( $J_t^I$ ).

The government bails out defaulted banks at a cost:

$$\text{bailout}_t = (1 - F_{\epsilon,t}^I) \left[ \epsilon_t^{I,+} - W_t^I + \eta\delta(1 - Z_{R,t}) (Z_{A,t}q_t^A A_t^I + Z_{M,t}q_t^M M_t^I) \right],$$

where  $\epsilon_t^{I,+} = E[\epsilon_t^I | \epsilon_t^I > V^I(W_t^I, \mathcal{S}_t^I)]$  is the expectation of  $\epsilon_t^I$  conditional on bankruptcy. Thus, the government absorbs the negative net worth of the defaulting banks. The last term are additional losses from bank bankruptcies, which are a fraction  $\eta$  of the mortgage assets and represent deadweight losses to the economy. The government bailout is what makes deposits risk-free, what creates deposit insurance.

**Government Debt.** To finance bailouts, the government issues risk-free short-term debt that trades at the same price as deposits. To service its debt, the government levies lump-sum taxes  $T_t^j$  on households of type  $j$  in period  $t$ , such that total tax revenue from lump-sum taxation is  $T_t = T_t^B + T_t^I + T_t^D$ . Therefore, if  $B_t^G$  is the amount of government bonds outstanding at the beginning of  $t$ , the government budget constraint satisfies

$$\bar{\pi}^{-1} B_t^G + \text{bailout}_t = q_t^f B_{t+1}^G + T_t. \quad (28)$$



Lump-sum taxes are levied in proportion to population shares and at a rate  $\tau_L$ :

$$T_t^j = \chi_j \tau_L \left( \bar{\pi}^{-1} B_t^G + \text{bailout}_t \right), \quad \forall j \in \{B, I, D\}. \quad (29)$$

This formulation ensures gradual repayment of government debt following a bailout.<sup>15</sup>

**REO Firm's Problem.** There is a continuum of competitive REO firms that are fully owned and operated by intermediary households (bank owners). Each period, REO firms choose how many foreclosed properties to buy from banks,  $I_t^{REO}$ , to maximize the NPV of dividends paid to intermediary households. The aggregate dividend in period  $t$  paid by the REO sector to the bank owners is:

$$D_t^{REO} = \underbrace{\left[ \rho_t + \left( S^{REO} - \nu^{REO} \right) p_t \right] K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO} I_t^{REO}}_{\text{REO investment}}. \quad (30)$$

The law of motion of the REO housing stock is:

$$K_{t+1}^{REO} = (1 - S^{REO}) K_t^{REO} + I_t^{REO}.$$

## 2.7 Depositor's Problem

The depositors' problem can also be aggregated, so that the representative depositor chooses nondurable consumption  $C_t^D$  and holdings of government debt and deposits  $B_t^D$  to maximize (2) subject to the budget constraint:

$$C_t^D \leq \underbrace{(1 - \tau) Y_t^D}_{\text{disp. income}} - \underbrace{\left( q_t^f B_{t+1}^D - \bar{\pi}^{-1} B_t^D \right)}_{\text{net deposit iss.}} - \underbrace{\nu^K p_t H_t^D}_{\text{own housing maint.}} - \underbrace{T_t^D}_{\text{lump sum taxes}}. \quad (31)$$

and a restriction that deposits must be positive:  $B_t^D \geq 0$ . Depositors consume their fixed endowment of housing services each period,  $H_t^D = \bar{K}^D$ .

<sup>15</sup>Equations (28) and (29) combined imply that new bonds issued in  $t$  are

$$B_{t+1}^G = \frac{1 - \tau_L}{q_t^f} \left( \bar{\pi}^{-1} B_t^G + \text{bailout}_t \right).$$

The case  $\tau_L = 1$  means that the government immediately raises taxes to pay for the complete bailout, and thus  $B_t^G = 0 \forall t$ . Any  $\tau_L < 1$  will generally imply a positive amount of debt outstanding, with the average debt balance decreasing in  $\tau_L$ . To ensure stationarity of the debt balance,  $\tau_L$  needs to be large enough relative to the average risk-free rate. We verify that this is the case in our quantitative exercises.

## 2.8 Financial Recessions

At any given point in time, the economy is either in a “normal” state, or a “crisis” state, the latter corresponding to a severe financial recession. This state evolves according to a Markov Chain with transition matrix  $\Pi$ . The financial recession state is associated with a higher value of  $\sigma_{\omega,t}$ , implying more idiosyncratic uncertainty; and a lower value of  $\zeta_t$ , implying a fall in aggregate house prices. Our financial recession experiments will feature a transition from the normal state into the crisis state alongside a low realization of the aggregate income shock  $\varepsilon_{y,t}$ .

## 2.9 Equilibrium

Given a sequence of endowment and crisis shock realizations  $[\varepsilon_{y,t}, (\sigma_{\omega,t}, \zeta_t)]$ , a competitive equilibrium is a sequence of depositor allocations  $(C_t^D, B_t^D)$ , borrower allocations  $(M_t^B, A_t^B, K_t^B, C_t^B, H_t^B, K_t^*, M_t^*, Z_{R,t}, \bar{\omega}_t^U)$ , intermediary allocations  $(M_t^I, A_t^I, K_t^{REO}, W_t^I, C_t^I, L_t^*, I_t^{REO}, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I)$ , and prices  $(r_t^*, q_t^M, q_t^A, q_t^f, p_t, p_t^{REO}, \rho_t)$ , such that borrowers, intermediaries, and depositors optimize, and markets clear:

$$\begin{aligned}
\text{New mortgages:} & \quad Z_{R,t} Z_{N,t} M_t^* = L_t^* \\
\text{PO strips:} & \quad \tilde{M}_t^I = \hat{M}_t^I \\
\text{IO strips:} & \quad \tilde{A}_t^I = \hat{A}_t^I \\
\text{Deposits and Gov. Debt:} & \quad B_{t+1}^I + B_{t+1}^G = B_{t+1}^D \\
\text{Housing Purchases:} & \quad Z_{R,t} Z_{N,t} K_t^* = S^{REO} K_t^{REO} + Z_{R,t} Z_{K,t} K_t^B \\
\text{REO Purchases:} & \quad I_t^{REO} = (1 - Z_{K,t}) K_t^B \\
\text{Housing Services:} & \quad H_t^B = K_t^B + K_t^{REO} = \bar{K}^B \\
\text{Resources:} & \quad Y_t = C_t^B + C_t^I + C_t^D + G_t \\
& \quad + \underbrace{\left(1 - F_{\epsilon,t}^I\right) \eta \delta (1 - Z_{R,t}) \left(Z_{A,t} q_t^A A_t^I + Z_{M,t} q_t^M M_t^I\right)}_{\text{DWL from bank failures}} \\
& \quad + \underbrace{\nu^K p_t (Z_{K,t} K_t^B + \bar{K}^I + \bar{K}^D) + \nu^{REO} p_t \left[K_t^{REO} + (1 - Z_{K,t}) K_t^B\right]}_{\text{housing maintenance expenditure}}
\end{aligned}$$

The resource constraint states that the endowment  $Y_t$  is spent on nondurable consumption, government consumption, deadweight losses from bank failures, and housing maintenance. Housing maintenance consists of payments for houses owned by borrowers, de-

positors, and intermediaries and for houses already owned by REO firms,  $K_t^{REO}$ , or newly bought by REO firms from foreclosed borrowers  $(1 - Z_{K,t})K_t^B$ . Government consumption consists of income taxes net of the mortgage interest deduction:

$$G_t = \tau(Y_t - Z_{A,t}A_t^B).$$

Appendix B contains an extensive discussion of the model's first order conditions.

## 2.10 Discussion of Key Model Assumptions

**Risky Mortgage Debt and Safe Asset Production.** One key friction in our model is that depositors only want to hold safe assets, but mortgages issued by borrowers are inherently risky. This creates the need for an intermediation sector that transforms the long-term mortgages with credit and pre-payment risk into short-term risk-free debt. Intermediaries use their equity capital to buffer mortgage losses. However, the intermediation sector only has a limited capacity to absorb losses, relying on the government as ultimate guarantor of the debt it issues. Thus, trade in debt claims between borrowers and savers (depositors) is subject to frictions stemming from the default options of both borrowers and banks. Borrower default causes foreclosures, which result in resource costs to society. Similarly, bank default causes costly liquidations, also resulting in the loss of resources. How a policy trades off these two margins is a key determinant of its resource efficiency.

**Allocation of House Price and Credit Risk.** Borrowers bear the majority of this risk with traditional fixed-rate mortgages, such that large drops in the aggregate house price cause a rise in foreclosures. Indexation of mortgage debt to house prices explicitly shifts house price risk to banks, potentially making them more fragile, while at the same time reducing borrower defaults and foreclosures. The contracts we consider implement a different allocation of risk on borrowers, intermediaries, and indirectly society at large due to the government guarantee of bank deposits.

A different possibility is that the government could directly take on house price risk, for example if the government-sponsored enterprises (GSEs) directly insured SAMs similarly to their current guarantee of conforming mortgages. We do not explore this possibility in our model, because we consider it unlikely that the government would seek direct exposure of its budget to large swings in house prices. Further, [Elenev et al. \(2016\)](#) and [Hurst, Keys, Seru, and Vavra \(2016\)](#) analyze issues with current GSE policy that would

likely be exacerbated by GSE insurance of SAMs. Yet another possibility is that banks would not directly hold SAMs on their balance sheets, but rather securitize these loans and sell them to investors. In the context of our model, these investors would be the intermediary households, since depositors do not participate in risky asset markets. However, in our model, intermediary households prefer to hold loans indirectly through banks, as this allows levered funding through guaranteed deposits. More generally, we view our assumption that indexation shifts risk to levered intermediaries with government guarantees as a sensible modeling approach. The boom and collapse in private-label securitization during the 2000s is a cautionary tale regarding banks ability (or desire) to shift the mortgage risk outside the levered financial system.

### 3 Calibration

This section describes the calibration procedure for key variables, and presents the full set of parameter values in Table 1. The model is calibrated at quarterly frequency and solved using global projection methods. Since the integrals (8) and (9) lack a closed form, we evaluate them using Gauss-Hermite quadrature with 11 nodes in each dimension.

**Exogenous Shock Processes.** Aggregate endowment shocks in (1) have quarterly persistence  $\rho_y = .977$  and innovation volatility  $\sigma_y = 0.81\%$ . These are the observed persistence and innovation volatility of log real per capita labor income from 1991.Q1 until 2016.Q1.<sup>16</sup> In the numerical solution, this AR process is discretized as a five-state Markov Chain, following the Rouwenhorst (1995) method. We abstract from long-run endowment growth ( $g = 0$ ). The average level of aggregate income (GDP) is normalized to 1. The income tax rate is  $\tau = 0.147$ , as given by the observed ratio of personal income tax revenue to personal income.

The discrete state follows a two-state Markov Chain, with state 0 indicating normal times, and state 1 indicating crisis. The probability of staying in the normal state in the next quarter is 97.5% and the probability of staying in the crisis state in the next quarter is 92.5%. Under these parameters, the economy spends 3/4 of the time in the normal state and 1/4 in the crisis state. This matches the fraction of time between 1991.Q1 and 2016.Q4

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<sup>16</sup>Labor income is defined as compensation of employees (line 2) plus proprietor’s income (line 9) plus personal current transfer receipts (line 16) minus contributions to government social insurance (line 25), as given by Table 2.1 of the Bureau of Economic Analysis’ National Income and Product Accounts. Deflation is by the personal income deflator and by population. Moments are computed in logs after removing a linear time trend.

that the U.S. economy was in the foreclosure crisis, and implies an average duration of the normal state of ten years, and an average duration of the crisis state of 3.33 years. These transition probabilities are independent of the aggregate endowment state. The low uncertainty state has  $\bar{\sigma}_{\omega,0} = 0.200$  and the high uncertainty state has  $\bar{\sigma}_{\omega,1} = 0.250$ . These numbers allow the model to match an average mortgage default rate of 0.5% per quarter in expansions and of 2.05% per quarter in financial recessions, which are periods defined by low endowment growth and high uncertainty. The unconditional mortgage default rate in the model is 0.95%. In the data, the average mortgage delinquency rate is 1.05% per quarter: 0.7% in normal times and 2.3% during the foreclosure crisis.<sup>17</sup>

**Local House Price Process.** We calibrate the persistence and variance of the local (insurable) housing quality process using FHFA house prices indices at the MSA level. Specifically, we run the annual panel regression

$$\log HPI_{i,t} = \delta_t + \phi_i + \rho_{\omega}^{ann} \log HPI_{i,t-1} + \varepsilon_{i,t} \quad (32)$$

where  $i$  indexes the MSA, and  $t$  indexes the year, and  $\delta_t$  and  $\phi_i$  are MSA and quarter fixed effects. The quarterly persistence is computed as  $\rho_{\omega} = (\rho_{\omega}^{ann})^{1/4}$ , which we estimate to be 0.977.<sup>18</sup> Since this persistence parameter only matters for the indexation of local house price risk, it is appropriate to calibrate this parameter only to local house price data. To calibrate  $\alpha$ , the share of house price variance at the local/regional level, we use (32) to compute the implied unconditional variance  $\text{Var}(\omega_{i,t}^L) = \text{Var}(\varepsilon_{i,t}) / (1 - (\rho_{\omega}^{ann})^2)$ , which delivers an unconditional standard deviation at the MSA level of 11.5%. We set  $\alpha = 0.25$ , which given our calibration for  $\sigma_{\omega,t}$  implies that the standard deviation of regional house prices is 10% in the model in normal times, and 12.5% in financial recessions, consistent with our empirical estimates.

**Demographics, Income, and Housing Shares.** We split the population into mortgage borrowers, depositors, and intermediary households as follows. We use the 1998 Survey

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<sup>17</sup>Data are for all residential mortgage loans held by all U.S. banks, quarterly data from the New York Federal Reserve Bank from 1991.Q1 until 2016.Q4. The delinquency rate averages 2.28% per quarter between 2008.Q1 and 2013.Q4 (high uncertainty period, 23% of quarters) and 0.69% per quarter in the rest of the period.

<sup>18</sup>The annual estimate is  $\rho_{\omega}^{ann} = 0.911$  with standard error 0.004 (clustered at the MSA level). The data source is the Federal Housing Finance Agency Quarterly All-Transactions House Price Index. The sample spans 1975.Q1 - 2017.Q1, and contains 13,649 observations drawn from 403 MSAs. The regression is run using an unbalanced panel as MSAs enter the sample over time, but results using a balanced panel limited to MSAs present since some given start date were nearly identical under a variety of start dates.

Table 1: Parameter Values: Baseline Calibration (Quarterly)

Parameter	Name	Value	Target/Source
<i>Technology</i>			
Agg. income persistence	$\rho_{TFP}$	0.977	Real per capita labor income BEA
Agg. income st. dev.	$\sigma_{TFP}$	0.008	Real per capita labor income BEA
Profit shock st. dev.	$\sigma_\epsilon$	0.070	FDIC bank failure rate
Transition: Normal $\rightarrow$ Normal	$\Pi_{00}$	0.975	Avg. length = 10Y
Transition: Crisis $\rightarrow$ Crisis	$\Pi_{11}$	0.925	25% of time in crisis state
<i>Demographics and Income</i>			
Fraction of borrowers	$\chi_B$	0.343	SCF 1998 population share LTV > .30
Fraction of intermediaries	$\chi_I$	0.020	Stock market cap. share of finance sector
Borr. inc. and housing share	$s_B$	0.470	SCF 1998 income share LTV > .30
Intermediary inc. and housing share	$s_I$	0.067	Employment share in finance
<i>Housing and Mortgages</i>			
Housing stock	$\bar{K}$	1	Normalization
Housing XS persistence	$\rho_\omega$	0.977	FHFA MSA-level regression
Housing XS dispersion (Normal)	$\bar{\sigma}_{\omega,0}$	0.200	Mortg. delinq. rate U.S. banks, no crisis
Housing XS dispersion (Crisis)	$\bar{\sigma}_{\omega,1}$	0.250	Mortg. delinq. rate U.S. banks, crisis
Local share of XS dispersion	$\alpha$	0.25	FHFA MSA-level regression
Inflation rate	$\bar{\pi}$	1.006	2.29% CPI inflation
Mortgage duration	$\delta$	0.996	Duration of 30-yr FRM
Prepayment cost mean	$\mu_\kappa$	0.370	Greenwald (2018)
Prepayment cost scale	$s_\kappa$	0.152	Greenwald (2018)
LTV limit	$\phi^K$	0.850	LTV at origination
Maint. cost (owner)	$\nu^K$	0.616%	BEA Fixed Asset Tables
<i>Intermediaries</i>			
Bank regulatory capital limit	$\phi^I$	0.940	Financial sector leverage limit
Deadweight cost of bank failures	$\eta$	0.085	Bank receivership expense rate
Maint. cost (REO)	$\nu^{REO}$	0.024	REO discount: $p_{ss}^{REO} / p_{ss} = 0.725$
REO sale rate	$S^{REO}$	0.167	Length of foreclosure crisis
<i>Preferences</i>			
Borr. discount factor	$\beta_B$	0.950	Borrower value/income, SCF
Intermediary discount factor	$\beta_I$	0.950	Equal to $\beta_B$
Depositor discount factor	$\beta_D$	0.998	3% nominal short rate (annual)
Risk aversion	$\gamma$	5.000	Standard value
EIS	$\psi$	1.000	Standard value
Housing preference (Normal)	$\bar{\xi}_0$	0.220	Borrower hous. expend./income
Housing preference (Crisis)	$\bar{\xi}_1$	0.160	HP growth volatility
<i>Government</i>			
Income tax rate	$\tau$	0.147	Personal tax rate BEA
Bailout taxation rate	$\tau_L$	1.0	Tractability

of Consumer Finances to define for every household a loan-to-value ratio. This ratio is zero for renters and for households who own their house free and clear. We define mortgage borrowers to be those households with an LTV ratio of at least 30%.<sup>19</sup> Those households make up for 34.3% of households ( $\chi_B = .343$ ). They earn 46.9% of labor income ( $s_B = .469$ ). For parsimony, we set all housing shares equal to the corresponding income share. Since the aggregate housing stock  $\bar{K}$  is normalized to 1,  $\bar{K}^B = .469$ .

To split the remaining households into depositors and intermediary households (bank owners), we set the share of labor income for bank owners equal to 6.7%. To arrive at this number, we calculate the share of the financial sector (finance, insurance, and real estate) in overall stock market capitalization (16.4% in 1990-2017) and multiply that by the labor income share going to all equity holders in the SCF. We set the housing share again equal to the income share. The population share of bank owners is set to 2%, consistent with the observed employment share in the FIRE sector. The depositors make up the remaining  $\chi_D = 63.7\%$  of the population, and receive the remaining  $s_D = 46.4\%$  of labor income and of the housing stock.

**Prepayment Costs.** For the prepayment cost distribution, we assume a mixture distribution, so that with probability 3/4, the borrower draws an infinite prepayment cost, while with probability 1/4, the borrower draws from a logistic distribution, yielding

$$Z_{R,t} = \Gamma_{\kappa}(\bar{\kappa}_t) = \frac{1}{4} \cdot \frac{1}{1 + \exp\left(\frac{\bar{\kappa}_t - \mu_{\kappa}}{\sigma_{\kappa}}\right)}$$

The calibration of the parameters follows [Greenwald \(2018\)](#), who fits an analogue of (43).<sup>20</sup> The parameter  $\sigma_{\kappa}$ , determining the sensitivity of prepayment to equity extraction and interest rate incentives, is set to that paper's estimate (0.152), while the parameter  $\mu_{\kappa}$  is set to match the average quarterly prepayment rate of 3.76% found in that exercise.

**Mortgages.** We set  $\delta = .99565$  to match the fraction of principal U.S. households amortize on mortgages.<sup>21</sup> The maximum loan-to-value ratio at mortgage origination is  $\phi^B =$

<sup>19</sup>Those households account for 88.2% of mortgage debt and 81.6% of mortgage payments.

<sup>20</sup>See [Greenwald \(2018\)](#), Section 4.2. The parameters are fit to minimize the forecast error  $LTV_t = Z_{R,t}LTV_t^* + (1 - Z_{R,t})\delta G_t^{-1}LTV_{t-1}$ , where  $LTV_t$  is the ratio of total mortgage debt to housing wealth,  $LTV_t^*$  is LTV at origination, and  $G_t$  is growth in house values.

<sup>21</sup>The average duration of a 30-year fixed-rate mortgage is typically thought of as about 7 years. This low duration is mostly the result of early prepayments. The parameter  $\delta$  captures amortization absent refinancing. Put differently, households are paying off a much smaller fraction of their mortgage principal

0.85, consistent with average standard mortgage underwriting norms.<sup>22</sup> Inflation is set to the observed 0.57% per quarter (2.29% per year) for the 1991.Q1 - 2016.Q4 sample.

**Banks.** We set the maximum leverage that banks may take on at  $\phi^I = 0.940$ , following [Elenev et al. \(2017\)](#), to capture the historical average leverage ratio of the leveraged financial sector. The idiosyncratic profit shock that hits banks has standard deviation of  $\sigma_\epsilon = 7.00\%$  per quarter. This delivers a bank failure rate of 0.33% per quarter, consistent with historical bank failure rate data from the FDIC.<sup>23</sup> We assume a deadweight loss from bank bankruptcies equal to  $\eta = 8.50\%$  of bank assets. This number falls in the interquartile range [5.9%,15.9%] of bank receivership expenses as a ratio of bank assets in a FDIC study of bank failures from 1986 until 2007 ([Bennett and Unal, 2015](#)). Deadweight losses from bank failures amount to 0.07% of GDP in equilibrium.

**Housing Maintenance and REOs.** We set the regular housing maintenance cost equal to  $\nu^K = 0.616\%$  per quarter or 2.46% per year. This is the average over the 1991-2016 period of the ratio of current-cost depreciation of privately-owned residential fixed assets to the current-cost net stock of privately-owned residential fixed assets at the end of the previous year (source: BEA Fixed Asset Tables 5.1 and 5.4).

We calibrate the maintenance cost in the REO state to  $\nu^{REO} = 2.40\%$  per quarter. It delivers REO housing prices that are 24.3% below regular housing prices on average. This is close to the observed fire-sale discounts reported by Fannie Mae and Freddie Mac during the foreclosure crisis. We assume that  $S^{REO} = 0.167$  so that 1/6th of the REO stock is sold back to the borrower households each quarter. It takes eight quarters for 75% of the REO stock to roll off. This generates REO crises that take some time to resolve, as they did in the data.

**Preferences.** All agents have the same risk aversion coefficient of  $\gamma_j = 5$  and intertemporal elasticity of substitution coefficient  $\psi = 1$ . These are standard values in the literature. We choose the value of the housing preference parameter in normal times

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than 1/7th each year in the absence of prepayment.

<sup>22</sup>The average LTV of purchase mortgages originated by Fannie and Freddie was in the 80-85% range during our sample period. However, that does not include second mortgages and home equity lines of credit. Our limit is a combined loan-to-value limit (CLTV). It also does not capture the lower down payments on non-conforming loans that became increasingly prevalent after 2000. [Keys, Piskorski, Seru, and Vig \(2012\)](#) document CLTVs on non-conforming loans that rose from 85% to 95% between 2000 and 2007.

<sup>23</sup>Based on the FDIC database of all bank failure and assistance transaction from 1991-2016, we calculate the asset-weighted average annual failure rate to be 1.65%.



$\bar{\xi}_0 = 0.220$  to match a ratio of housing expenditure to income for borrowers of 18%, a common estimate in the housing literature.<sup>24</sup> The model produces a ratio of 17.5%. To induce an additional house price drop, we set  $\bar{\xi}_1 = 0.16$  in the crisis states. This additional variation yields a volatility of quarterly log national house price growth of 1.41%, compared to 1.66% in the data (source: Case Shiller national home price index, deflated by PCE, 1991.Q1 - 2016.Q4).

For the time discount factors, we set  $\beta^B = \beta^I = 0.950$  to target the ratio of housing wealth to quarterly income for borrowers of 8.57, close to the same ratio for “borrowers” as defined above in the 1998 SCF (8.67). Finally, we set the discount rate of depositors  $\beta^D = 0.998$  to match the observed nominal short rate of 2.9% per year or 0.71% per quarter. With these parameters, the model generates average borrower mortgage debt to housing wealth (LTV) of 64.5%, close to the corresponding value 61.6% for the “borrower” population in the 1998 SCF.

**Government.** We set the income tax rate  $\tau$  in the model to match the average effective personal tax rate of 14.7% as reported by the BEA. We further set the fraction of bailout expenses funded through lump-sum taxation in the same period,  $\tau_L$ , to 100%. This assumption guarantees that the outstanding balance of government debt  $B_t^G$  is always zero, which avoids government debt as state variable. In Section 5, we test the sensitivity of our quantitative conclusions to a different taxation regime with a positive amount of government debt. We find that the assumption of instantaneous taxation does not significantly affect our quantitative conclusions about the different indexation schemes.

## 4 Main Results on Mortgage Indexation

Our main exercise compares the economy with regular mortgages to hypothetical economies with varying degrees and forms of mortgage indexation. Specifically, we solve: (i) a No Index model with  $\iota_p = \iota_\omega = 0$  (the benchmark); (ii) an Aggregate (only) model with  $\iota_p = 1$  and  $\iota_\omega = 0$ ; (iii) a Local (only) model, with  $\iota_p = 0$  and  $\iota_\omega = 1$ ; (iv) and a Regional model that combines aggregate and local indexation, with  $\iota_p = 1$  and  $\iota_\omega = 1$ . We conduct a long simulation for each model, and display the resulting averages of key prices and

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<sup>24</sup>Piazzesi, Schneider, and Tuzel (2007) obtain estimates between 18 and 20 percent based on national income account data (NIPA) and consumption micro data (CEX). Davis and Ortalo-Magné (2011) obtain a ratio of 18% after netting out 6% for utilities from the median value of 24% across MSAs using data on rents.

quantities in Table 2.

These stylized experiments are designed to showcase the different properties of aggregate and local indexation. While the typical SAM proposal does not distinguish between the source of house price movements, any indexation scheme can be decomposed into these two types. We show that these forms of indexation yield sharply different economic implications, which should be considered when designing a mortgage product.

**Benchmark: Unconditional Moments.** Before turning to the indexation results, it is useful to briefly discuss the benchmark model. On the borrower side, the model generates average mortgage debt to annual income of 64.9%, close to the observed value of 69%. It generates an aggregate LTV ratio among mortgage borrowers of 64.5%, close to the value of 67.2% in the data. The average mortgage default rate of 0.95% per quarter matches the data, and the loss-given-default rate of 38.61% comes close to the data. The implied loss rate is 0.40% per quarter. The refinancing rate of 3.84% per quarter matches the implied average rate at which mortgages are replaced excluding rate refinances. The maximum LTV constraint, which only applies at origination and caps the LTV at 85% always binds in our simulations, consistent with the overwhelming majority of borrowers taking out loans up to the limit.

On the intermediary side, we match the leverage ratio of the levered financial sector, which is 93.98% in the model. Banks' regulatory capital constraints bind in 99.35% of the periods. Bank equity capital represents about 4.6% of annual GDP (18.4% of quarterly GDP) and 7.09% of bank assets in the model. Bank deposits (that go towards financing mortgage debt) represent just over 61.4% of annual GDP (245.4%/4). Bank dividends are 1.0% of GDP. The model generates a substantial amount of financial fragility. One measure thereof is the bank default rate. In the benchmark, it is 0.33% per quarter or 1.3% per year. Deadweight losses from bank bankruptcies are 0.07% of GDP on average.

The REO firms represent the other part of the intermediary sector. They spend 0.34% of GDP on housing maintenance on average, and pay 0.5% of GDP in dividends to their owners. REO firms earn very high returns from investing in foreclosed properties and selling them back to the borrowers: the return on equity is 5.3% per quarter (equal to the return on assets since the REO firms have no leverage).<sup>25</sup>

The model somewhat overstates housing wealth, which represents about 220.3% of annual GDP in the model and 153% in the data. This discrepancy is an artifact of giving

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<sup>25</sup>This return on equity in the model mimics the high returns earned by single-family rental firms like Blackstone's Invitation Homes over the past five years.

all agents the same housing to income ratio in the model, while the “borrower” type holds relatively more housing in the data than the other groups. At equilibrium, only borrower holdings of housing are relevant,<sup>26</sup> so the quantitative effect of exaggerating total housing wealth is minimal. The mortgage rate exceeds the short rate by 75bps per quarter, which is close to the average spread between the 30-year fixed-rate mortgage rate and the 3-month T-bill rate of 89bps per quarter for 1991–2016. The model’s expected excess return, or risk premium, earned by banks on mortgages is 34bps per quarter.

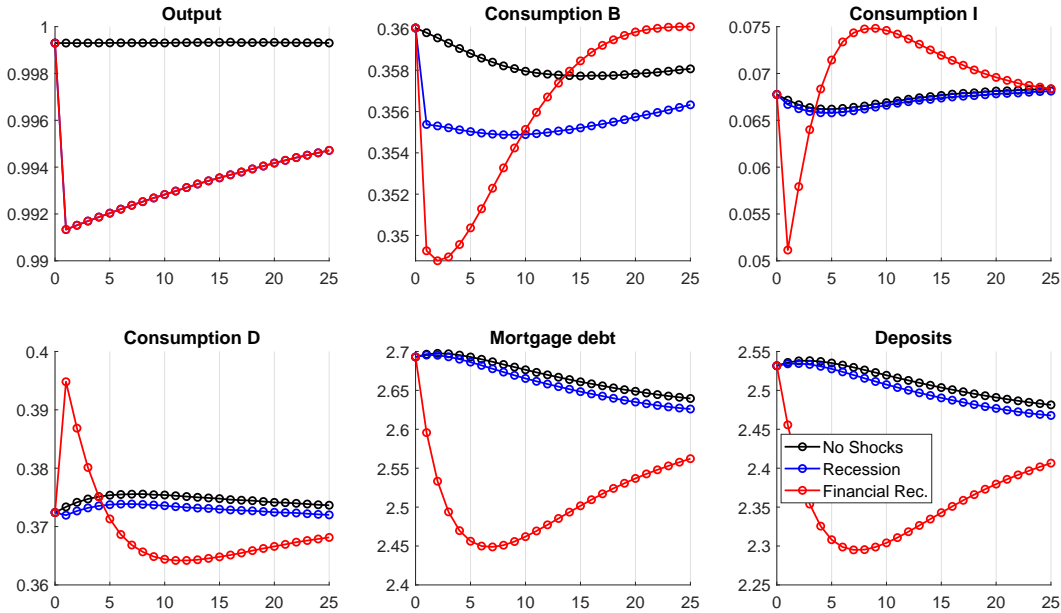
**Benchmark: Financial Crises.** To understand risk-sharing patterns in the benchmark economy, it is instructive to study how this economy behaves in a financial recession and a non-financial recession. We define a non-financial recession event as a one standard deviation drop in aggregate income while the economy is in the normal (non-crisis) state. In a financial recession, the economy experiences the same fall in income, but also transitions from the normal state into the crisis state, leading to an increase in house value uncertainty ( $\bar{\sigma}_{\omega,0} \rightarrow \bar{\sigma}_{\omega,1}$ ) and a decrease in housing utility ( $\bar{\xi}_0 \rightarrow \bar{\xi}_1$ ). We simulate many such recessions to average over the endogenous state variables (i.e., the wealth distribution). Figures 1 and 2 plot the impulse-response functions in levels, with financial recessions indicated by red circles and non-financial recessions in blue.<sup>27</sup> By construction, the blue and red lines coincide in the top left panel of Figure 1.

A financial crisis results in a significant increase in mortgage defaults. The risk on existing mortgages goes up, but the fixed interest rates do not, causing the value of bank assets to fall. Faced with reduced equity, some banks fail, while the remaining ones are forced to delever in the wake of the losses they suffer, substantially shrinking both mortgage assets and deposit liabilities. To induce depositor households to reduce deposits and increase consumption, the real interest rate falls sharply. Intermediary consumption falls heavily, as the owners of the intermediary sector absorb losses from their banks. Borrower consumption also falls as borrowers cut back on new mortgage borrowing, and must help pay for the bank bailouts by paying higher taxes. After the shock, the economy gradually recovers as high excess returns on mortgages eventually replenish bank equity.

<sup>26</sup>We could set the housing to income ratios of intermediaries and depositors to match the overall housing to GDP ratio observed in the data. However, the constant housing capital of these two types of households only affects their equilibrium non-durable consumption levels through housing maintenance payments. The effect of such an adjustment on equilibrium outcomes is negligible.

<sup>27</sup>The simulations underlying these generalized IRF plots are initialized at the ergodic distribution of the endogenous states, the mean income level, and in the non-crisis state ( $\bar{\sigma}_{\omega,0}, \bar{\xi}_0$ ).

Figure 1: Financial vs. Non-financial Recessions: Benchmark Model (part 1)



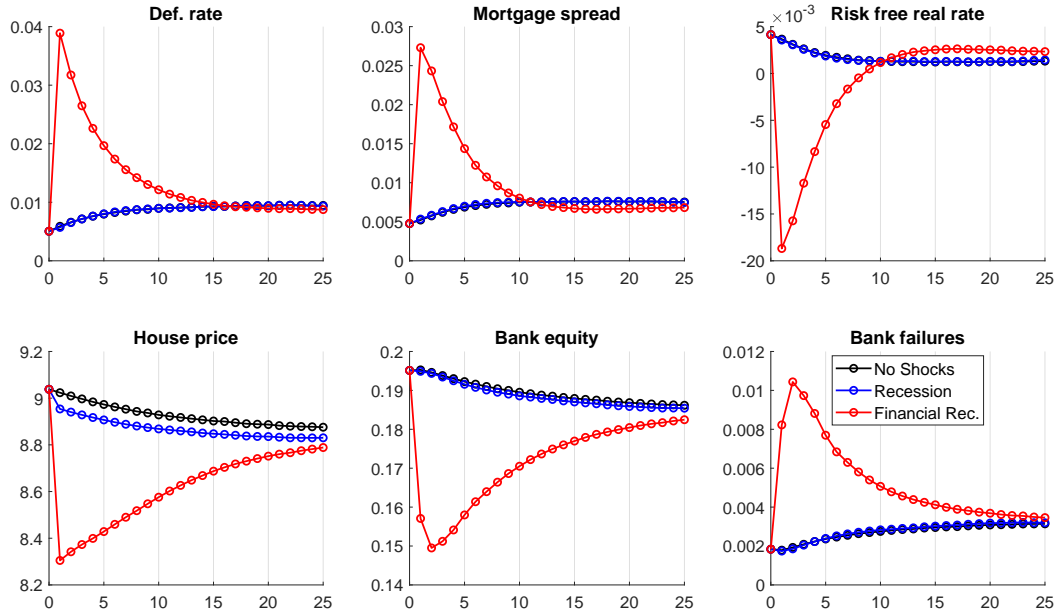
**Black:** no shocks, **Blue:** non-financial recession, **Red:** financial recession.

**Aggregate Indexation.** The first experiment we consider is one where all mortgage payments are indexed to aggregate house prices. The conjecture in the literature is that this should reduce mortgage defaults and generally improve borrower’s ability to smooth consumption. Perhaps surprisingly, we find that this conjecture does not hold up in general equilibrium. To the contrary, Table 2 shows that, by adding to financial fragility, aggregate indexation destabilizes borrower consumption while leaving mortgage default rates unchanged.

To understand this, we can turn to Figure 3, which compares financial recessions in the benchmark and Aggregate models. Under aggregate indexation, banks find themselves exposed to increased risk through their loan portfolio, whose cash flows now fluctuate directly with aggregate house price movements. Although banks optimally choose to hold a slightly larger capital buffer compared to the benchmark model, this is insufficient to protect their equity from the much greater risks they face. Left with a trade-off between preserving charter value and exploiting limited liability, banks lean more toward their option to declare bankruptcy and saddle the government with the losses.

The combination of increased risk and the absence of precautionary capital means that the share of bank defaults upon entering a financial recession is vastly larger in the Aggregate economy — with nearly 40% of banks failing — relative to the No Index benchmark.

Figure 2: Financial vs. Non-financial Recessions: Benchmark Model (part 2)



**Black:** no shocks, **Blue:** non-financial recession, **Red:** financial recession.

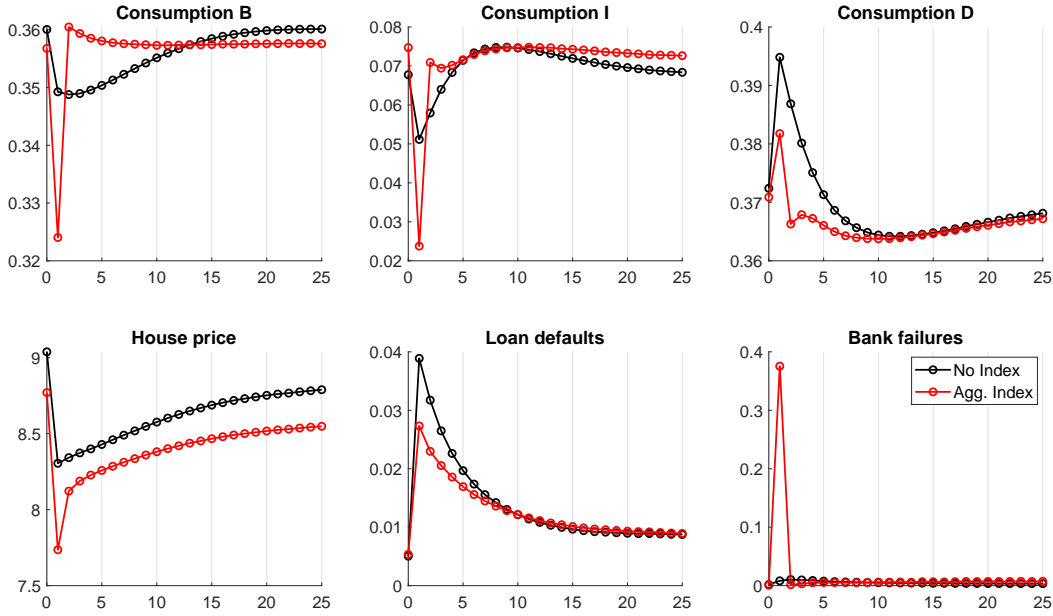
This spike in bank failures necessitates a wave of government bailouts of bank deposits, placing a large tax burden of 8% of GDP on the population. This tax obligation depresses borrower consumption and housing demand, leading to a larger drop in house prices relative to the benchmark. The breakdown in intermediation and risk sharing is reflected in the upward spike in depositor consumption while at the same time borrower and intermediary households have to sharply cut consumption.<sup>28</sup>

Aggregate indexation provides a modest reduction in mortgage default in the financial recession. Although this indexation protects borrowers from the large fall in national house prices, it is unable to stave off the increase in defaults due to higher idiosyncratic dispersion  $\sigma_{\omega,t}$ . Importantly, aggregate indexation is indiscriminately targeted, providing equal relief to the hardest-hit and relatively unaffected regions/households alike, limiting its effects on the number of foreclosures.

The bottom half of Table 2 compares welfare and consumption outcomes across the different indexation regimes. The increased financial fragility results in incredibly volatile intermediary wealth ( $W^I$  growth volatility goes up 1366.8%). Intermediary consump-

<sup>28</sup>In Section 5 we allow the government to fund the bailout expenditure mainly through issuing government debt. There we confirm that these crisis dynamics do not depend on the assumption of immediate taxation, but rather are a result of the breakdown in mortgage credit.

Figure 3: Financial Recessions: Benchmark vs. Aggregate Model



**Black:** benchmark financial recession, **Red:** aggregate index. financial recession. Responses are plotted in levels.

tion is more unconditionally more volatile ( $C^I$  growth volatility goes up 392.9%), and falls more in a financial recession. Borrower consumption growth volatility increases by 351.3%, albeit from a much lower base. These results point to a deterioration in risk sharing between borrowers and intermediaries under aggregate indexation, further evidenced by a rise of 145.7% in the volatility of the log marginal utility ratio between these types (row 29).

To assess the gains from aggregate indexation, we aggregate agents' value functions to obtain measures of welfare.<sup>29</sup> Borrowers are made worse off (row 18), both because their consumption has become more volatile (row 24) and because their consumption is lower (row 21) for reasons explained above. Borrowers face lower house prices and higher mortgage rates. Depositors' welfare (row 19) is essentially unchanged. Their mean consumption (row 22) is slightly lower, mostly because they earn lower interest rates on their savings, accumulating less wealth as a result (row 11). However, their consumption also becomes less volatile (row 25), causing a neutral net effect.<sup>30</sup>

<sup>29</sup>There are many ways of computing aggregate welfare in incomplete markets economies with heterogeneous agents. The measure we present calculates welfare per capita for each agent type, multiplies it by the population share of each type, and sums across types.

<sup>30</sup>Depositor consumption becomes less volatile because it experiences a smaller spike in financial recessions, see also Figure 3. This smaller spike is due to higher taxes that need to be raised to cover losses from

Finally, and perhaps surprisingly, intermediary households are made better off. Intermediary consumption levels increase because of the higher returns they earn on mortgage assets and REO housing, and because of an increase in the value of banks' default option, allowing for very high consumption in good times but limited downside in bad times. This positive effect on the average level of consumption outweighs the utility loss from the massive increase in intermediary consumption volatility. All told, we obtain the interesting distributional result that insuring borrower exposure to aggregate house price risk leads bank owners to gain at the expense of both borrowers and savers.

**Local Indexation.** Next, we can turn to the Local economy ( $\iota_p = 0, \iota_\omega = 1$ ), displayed in the third column of Table 2, which indexes only to the local component of house values, while ignoring the aggregate component. In practice, such a contract would be implemented by subtracting an aggregate house price index from regional indexes, and then indexing the debt of local borrowers to only the local residual. For example, during the Great Recession house prices fell substantially more in Las Vegas than in Boston. Purely local indexation would have implied a reduction in mortgage debt for Las Vegas borrowers, but an increase in debt for Boston borrowers.

In sharp contrast to aggregate indexation, indexing mortgage debt to relative local house prices (only) stabilizes the financial sector while substantially reducing the frequency of borrower default. To begin, we can turn to Figure 4, which compares crises in the benchmark model to crises in a model with only partial local indexation. Although borrowers must absorb a similar fall in aggregate house prices as in the baseline, local indexation is still largely successful at reducing foreclosures, sending targeted debt relief to households in areas where house prices fell the most.<sup>31</sup> Unlike in the aggregate indexation case, the reduction in defaults under local indexation is not accompanied by large losses from indexation, since the (perfectly diversifiable) local shocks wash out in aggregate. As a result, the rate of bank failures is *lower* in a crisis under local indexation, a sign of improved financial stability.

Turning to unconditional moments in the third column of Table 2, we observe that the average borrower default rate falls precipitously, with a reduction of nearly half relative

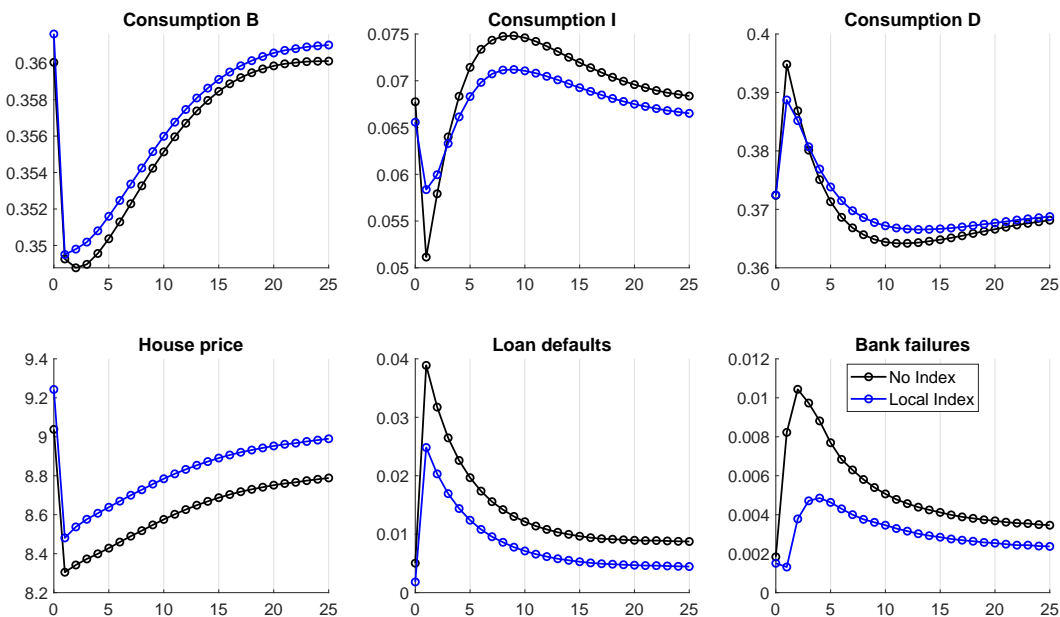
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bank bailouts.

<sup>31</sup>For intuition, recall that the average borrower in the model, similar to the data, has typical leverage around 66%. Thus, the typical borrower could absorb a very large fall in aggregate house prices (on the order of the 2008 housing crash) and still remain above water. Instead, the typical defaulting borrower must *also* receive an adverse local or idiosyncratic shock. Effectively indexing against these shocks is therefore a potent force against default, even during an aggregate house price decline.

to the benchmark. While aggregate and local indexation are roughly equally effective at reducing default in a financial crisis, which is largely driven by aggregate house prices, local indexation is much more effective than aggregate indexation in normal times, when default is primarily driven by local and idiosyncratic shocks. Facing less default risk, banks lower mortgage interest rates, pushing up house values. These higher values support increased household borrowing, raising the average stock of mortgage debt, in turn financed with a larger deposit base. While banks react to this reduced risk by holding as little capital as allowed, the required minimum is sufficient to ensure a large decrease in the rate of bank failures. The risk-free interest rate rises slightly as the supply of deposits expands to meet the demands of a larger intermediation sector. At the same time, lower mortgage risk is reflected also in lower mortgage risk premia and mortgage spreads. Overall, the banking system is both safer and larger under this contract, but receives less compensation on a per-loan basis.

Figure 4: Financial Recessions: Benchmark vs. Local Model



**Black:** benchmark financial recession, **Blue:** local index. financial recession. Responses are plotted in levels.

The welfare effects from local indexation are the reverse of those from aggregate indexation. Borrowers and depositors gain while intermediaries lose. Risk sharing in the economy improves dramatically, as the volatility of marginal utility ratios between groups falls, especially between borrowers and intermediaries (rows 39, 40). Depositors and intermediaries also see large reductions in consumption growth volatility, while borrowers



experience slightly increased volatility — albeit from a low level — due to larger housing and mortgage positions.<sup>32</sup>

In sum, indexation to local house price shocks is highly effective at reducing the risk of foreclosures and financial fragility. More intermediation ensues, which makes both borrowers and savers richer. However, banking becomes less profitable.

**Regional Indexation.** The fourth column of Table 2 shows results for the Regional model, which indexes to both aggregate and local house price variation. Unsurprisingly, the simulation means in this column mostly lie between the Aggregate and Local cases in columns 2 and 3. While pairing local and aggregate indexation decreases the bank default rate in the Regional model relative to the Aggregate model, the destabilizing effect of aggregate indexation is still enough to increase bank defaults relative to the No Index specification. The high degree of indexation in this economy strongly reduces the incentives to default, leading to the lowest borrower default rates among these four specifications. Nonetheless, signs of financial instability remain, particularly in the high consumption and wealth growth volatilities of the intermediary. Aggregate welfare is 0.32% higher in the Regional model relative to the No Index model.

## 5 Extensions

**Interest vs. Principal Indexation.** So far, our indexation applied both to interest payments and to principal. However, a number of the contract proposals mentioned in the literature review envision indexing interest payments only, while leaving principal balances unchanged. These proposals are motivated by e.g., work by [Fuster and Willen \(2015\)](#) and [Di Maggio et al. \(2017\)](#) who suggest that households respond strongly to interest payment adjustments, as well as work by e.g., [Ganong and Noel \(2017\)](#) showing that households barely respond to principal adjustments, at least when the latter leave them underwater. To investigate these contracts, as well as the role of indexing each component of the mortgage payment more generally, we run a series of experiments in which either interest or principal payments, but not both, are indexed to house prices. The corresponding default thresholds are derived in Appendix C.1.

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<sup>32</sup>The smaller changes in intermediary and depositor consumption during crises (top row of Figure 4) underscore this point. Depositors earn higher interest rates under this system, while borrowers pay lower rates on their mortgages, helping to boost the consumption of each group. In contrast, intermediary households' mean consumption falls by 2.9% as dividends from REO firms and banks decline.

The first four columns of Table 3 contrast the No Index and Regional models with Reg-IO and Reg-PO specifications that index only interest and principal payments, respectively. Interestingly, imposing *either* Reg-IO or Reg-PO indexation in isolation *lowers* bank default rates (0.30% and 0.31%, respectively) relative to the No Index model (0.33%), even though applying both types of indexation simultaneously in the Regional model substantially increases bank failures (0.50%). This points to an interesting nonlinearity, where a moderate amount of indexation improves financial stability, while too much indexation — particularly aggregate indexation — clearly harms it.

Turning to borrower default, we see that indexing principal only delivers nearly all the default reduction in the Regional model, while indexing interest only gives up much of these gains. The key difference is that, under fixed-rate mortgages, changes in interest payments last only until the next loan renewal, which occurs every 6-7 years on average in the model, while changes in principal balance will influence payments even after the loan has been renewed. As a result, indexing principal has a much larger impact on continuation costs and default decisions than indexing interest payments. By providing effective protection from default, but shielding banks from present-value losses on interest payments, the Reg-PO model yields substantially higher levels of mortgage debt, deposits, and house prices relative to the Regional model. In contrast, indexing interest payments only delivers higher house prices relative to the Regional model, but no increase in debt, due to an increase in credit spreads.

Turning to welfare, we find that, as in the Regional model, borrowers benefit in both the Reg-IO and Reg-PO cases relative to the benchmark economy. These economies deliver higher average consumption to borrowers, who finance fewer bank bailouts, without generating the large increase in borrower consumption volatility found in the Regional model. The depositors' value function decreases modestly relative to the benchmark in both cases, as lower average consumption due to higher maintenance payments on more valuable houses outweighs the benefit of less volatile consumption.

Intermediaries still suffer large welfare losses in the Reg-IO and Reg-PO economies relative to the No Index case. Under the Reg-IO model, a large drop in the volatility of intermediary wealth and consumption cannot make up for the loss of intermediation profits due to lower spreads and fewer bailouts, results akin to those we found with local indexation. In Reg-PO case, intermediary wealth and consumption are actually more volatile than in the benchmark model, while spreads are far below those of the benchmark. This additional intermediary instability in the Reg-PO relative to the Reg-IO model is intu-

itive, since in the former case banks absorb much larger present value changes (through indexing the entire mortgage principal) when prices move.

**Asymmetric Indexation.** Many real-world SAM proposals envision reducing mortgage payments when house prices fall but not increasing payments when prices rise. We now introduce such asymmetric contracts in the Regional model. As before, we assume indexation to aggregate and local house price components, but we now cap the maximum upward indexation in each dimension. With asymmetric indexation, our assumption of i.i.d. house quality shocks  $\omega_{i,t}$  is no longer equivalent to more realistic persistent  $\omega_{i,t}$  processes. To address this, we now model the  $\omega_{i,t}^L$  and  $\omega_{i,t}^U$  shocks as AR(1) processes for the results below. Appendix C.2 provides details on this extension and the corresponding optimality conditions.

Column 5 of Table 3 presents the results for the Reg-Asym case, demonstrating that asymmetric indexation substantially alters the mortgage landscape. To begin, banks now expect to take losses on average from indexation, since the debt relief they offer on the downside is no longer compensated by higher debt repayments when house prices increase. As a result, banks set much higher mortgage rates ex-ante, equal to 2.37% per quarter, to compensate them for these asymmetric transfers back to the households. At the aggregate, this has an effect very similar to shortening the amortization schedule of the bond (reducing  $\delta$ ), since borrowers make higher coupon payments in exchange for a much larger effective principal reduction each period — albeit occurring largely through indexation rather than explicit principal payments. House prices also fall, as the collateral value of housing is lower under the effectively more front-loaded and therefore less desirable asymmetric indexation contracts. Lower house prices imply lower mortgage balances, lower deposits, and a smaller financial sector overall.

Although borrowers partially compensate for the higher mortgage rates by increasing the average refinancing rate, the faster effective amortization of these loans leads to much lower household leverage, as the large increase in principal forgiveness overwhelms the higher rate of new borrowing. Lower household leverage in turn virtually eliminates default, since it now takes much larger shocks to push borrowers underwater. Nonetheless, financial fragility is massively increased under this contract system, as the financial sector losses from indexation in bad times are no longer fully offset by increased payments when prices rise. The total loss rate for banks is 0.91%, several times larger than in the No Index and Regional models. Faced with this increase in losses, banks increase their eq-

uity capital buffer modestly, but not by nearly enough to undo their additional risk. As a result, banks fail much more frequently, laying off the increased risk largely onto taxpayers. The deadweight losses from bank defaults increase by 70% relative to the symmetric indexation case, reducing resources available for consumption. The increase in financial fragility can also be seen in the severe increase in the volatility of intermediary wealth and consumption, on the order of the model with aggregate indexation only.

Turning to welfare, intermediaries suffer large losses, not only due to an increase in volatility, but also due to a drop in average consumption, as the much smaller financial system reduces intermediation income. This stands in contrast to our earlier finding with symmetric indexation that financial fragility tends to be good for intermediaries. Although borrower consumption becomes more volatile, borrowers are better off with asymmetric indexation due to an increase in average consumption, largely due to a fall in maintenance costs as house prices fall (an assumption relaxed below).<sup>33</sup> The aggregate welfare gain in the Reg-Asym model is 0.73%.

Column 6 of Table 3 presents the asymmetric indexation of interest payments only (Asym-IO), leaving the principal balance and principal payments unindexed, which arguably comes closest to the real world proposals. Once again, banks anticipate substantial net forgiveness to borrowers, causing a rise in the mortgage rate, although not as extreme as in the Reg-Asym case. Household leverage once again falls, in part for the same reasons as in the Reg-Asym case.<sup>34</sup> All told, this lower leverage reduces mortgage defaults to 0.55% per quarter, on the order of the Regional model, but falling short of the Reg-Asym case.

Reducing the financial sector exposure to indexation losses on principal leads to a lower bank failure rate relative to the Reg-Asym case, but still a much higher one than in the Reg-IO case. Similarly, the mortgage market and banking sector do not shrink as much, nor do house prices fall as much. While borrowers gain by less than in the Reg-

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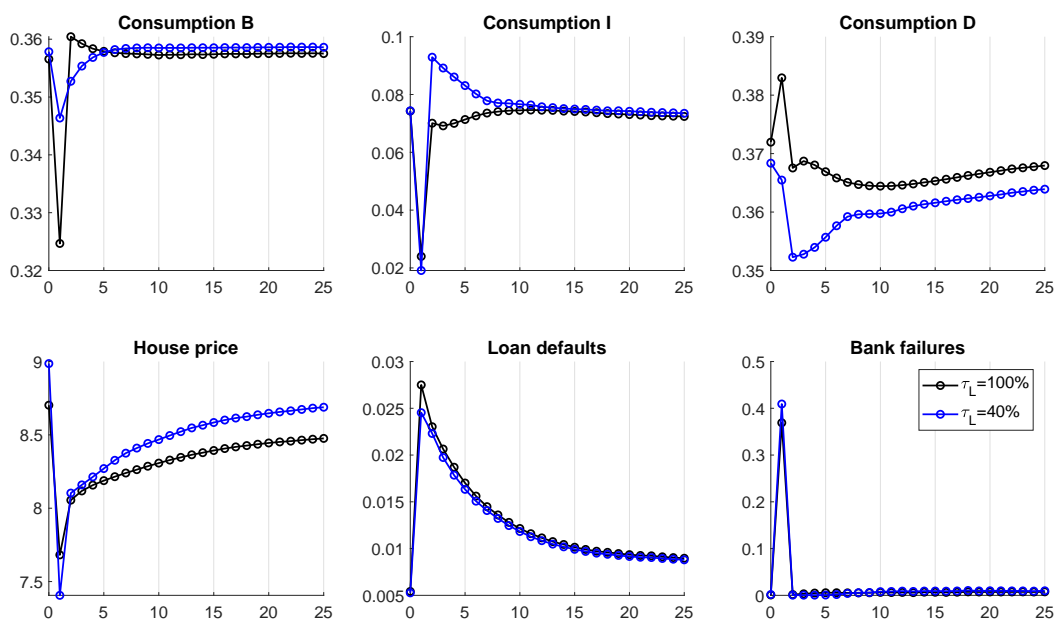
<sup>33</sup>This welfare improvement is also due in part to the fact that under a lower level of steady state debt, borrowers make lower debt service payments. As a result, this welfare comparison between two steady states may overstate the benefits to borrowers, since it ignores the painful deleveraging period borrowers must undergo to get to the new steady state. Indeed, Table 4 in Appendix C.3 shows that borrower consumption initially falls along the transition path due to deleveraging. Nonetheless, the welfare benefit for borrowers *including* the transition path, is a similar 1.76%, again largely due to a fall in maintenance expenses under lower house values.

<sup>34</sup>This fall is strengthened by an additional and interesting force. Because interest payments are asymmetrically indexed, but principal payments are not, the average interest rate on loans as they age is falling. This tends to make older loans, many of which have experienced some interest forgiveness, favorable to new loans issued at the “full” interest rate. This pushes down the incentive to refinance, reducing the average refinancing rate. Less frequent refinancing of loans implies a longer period of repayment and forgiveness between renewals, leading to lower leverage.

Asym indexation regime, intermediaries find the Asym-IO system to be even worse than the Reg-Asym version, since it reduces intermediation profits without providing enough volatility for intermediaries to take full advantage of their limited liability option. The welfare gain in the Asym-IO model is 0.25%.

**Government Debt.** Our calibration assumes that the government raises lump-sum taxes to fully pay for any bailout of depositors within each period. As a result, when a large fraction of banks fail, taxes required to fund the bailout can sharply reducing borrower and intermediary consumption, most notably in the aggregate indexation model (Figure 3). This immediate tax burden might be smaller if the government financed bailouts with debt, potentially reducing the severity of financial recessions. To test the sensitivity of crisis dynamics to different taxation regimes, we solve the Aggregate model with a much higher degree of tax smoothing. We set  $\tau_L = 0.4$ , implying that each period, the government uses taxes to pay 40% of its outstanding liabilities (past debt plus expenses for current bailouts), with the remainder funded by new debt. Figure 5 compares crisis dynamics in the Aggregate model in this smoothed tax specification to the Aggregate model with immediate taxation ( $\tau_L = 1$ ).

Figure 5: Financial Recessions: Full vs. Partial Taxation for Bailout Funding (part 1)



**Black:** financial recession in aggregate indexation baseline, **Blue:** financial recession with aggregate indexation and tax smoothing. Responses are plotted in levels.

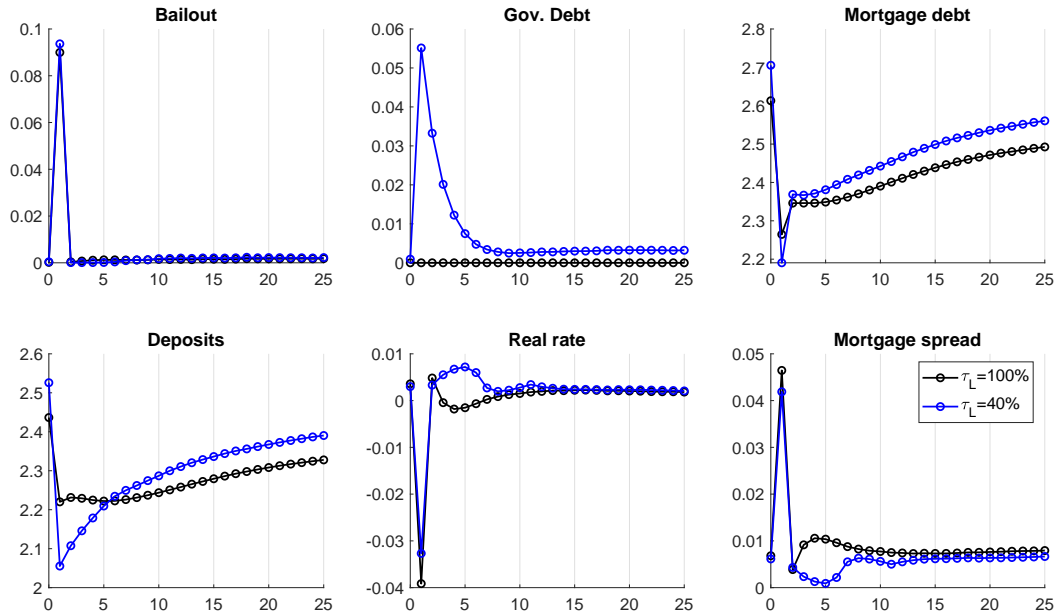
Two main conclusions emerge. First, consumption dynamics in the crisis change with the introduction of government debt. Second, tax smoothing is, if anything, worse for financial fragility. Starting on the consumption side, borrower consumption falls by much less on impact, as the tax burden is postponed further into the future. However, depositor consumption is substantially reduced, as they must purchase government debt to fund the bailout. To induce the depositors to absorb this debt, the real risk-free interest rate, which is both the deposit rate and the yield on government debt, needs to increase compared to the immediate-taxation model. At this higher real rate, banks issue fewer deposits. The quantity of deposits falls by 20% in the smooth-taxation economy, compared to 10% in the immediate-taxation case, as government safe asset provision crowds out private safe asset production (Azzimonti and Yared, 2018). The real rate stays elevated for 5 quarters after the initial shock, increasing banks funding cost and compressing mortgage spreads and intermediary consumption. Banks respond to the lower supply of deposits by cutting their lending more sharply, reducing the availability of mortgage credit to borrowers, leading to a much sharper drop in house prices. At the same time, higher funding costs reduce bank net worth, increasing the rate of bank failures.

The comparison demonstrates that the severe crisis dynamics in financial recessions with Aggregate are not an artifact of the instantaneous taxation of borrowers and intermediaries. Instead, the primary driver of the steep drop in house prices is the sharp contraction in the size of the financial sector. This contraction is only amplified when bailouts are funded through government debt since the higher cost of deposit funding causes a larger decline in lending.

**Liquidity Defaults.** A potential concern with our approach is that in reality, many mortgage defaults are triggered — at least in part — by household liquidity shocks, while our model only considers strategic default. Appendix C.4 extends the model to allow for a combination of liquidity and strategic defaults. Households receive a liquidity shock, in which case they sell if above water and default if under water. We also impose a utility penalty for default, which discourages strategic default unless borrowers have sufficiently negative home equity. In this extension, we calibrate to a 55% share of defaults that are liquidity defaults.

Overall, the model with liquidity default generates very similar conditional and unconditional moments to our baseline with only strategic default. The welfare comparison between the No Index, Aggregate, Local, and Regional economies is both qualitatively

Figure 6: Financial Recessions: Full vs. Partial Taxation for Bailout Funding (part 2)



**Black:** financial recession in aggregate indexation baseline, **Blue:** financial recession with aggregate indexation and tax smoothing. Responses are plotted in levels.

and quantitatively unaffected. Although liquidity default is determined mechanically by whether not a borrower is above water, while strategic default is determined by a comparison of present value costs and benefits, both are driven primarily by household leverage, and yield similar dynamics. Generating substantially different behavior from liquidity defaults would require a large fraction of *above-water* foreclosures, an assumption that is not supported by the data. In short, our findings are robust to this source of borrower defaults.

**Maintenance Costs.** The baseline model assumes that housing maintenance is proportional to the current house price, capturing the stylized fact that residential investment increases with house prices. Since housing maintenance is a use of resources that could be consumed, our indexation experiments can influence consumption by changing the level of house prices. To make sure that this assumption is not driving our main findings, Appendix C.5 considers a model where housing maintenance is rebated lump-sum to households. While our main results are largely similar, the Reg-Asym model — which generated the largest drop in house prices — now yields a lower welfare gain than the Regional model.

## 6 Conclusion

Redesigning the mortgage market through product innovation may allow an economy to avoid a severe foreclosure crisis like the one that hit the U.S. economy in 2008-2010. To this end, we study the implications of indexing mortgage payments to aggregate or local house prices in a general equilibrium model with incomplete risk-sharing, costly default, and a rich intermediation sector.

A key finding is that indexing mortgage debt to aggregate house prices may increase financial fragility. Inflicting large losses on highly-levered lenders in bad states of the world can cause systemic risk (high bank failure rates), costly taxpayer-financed bailouts, larger house price declines, and higher risk premia on mortgages, all of which ultimately hurt the borrowers the indexation was intended to help. Moreover, aggregate indexation redistributes wealth from borrowers and depositors towards bank owners, since a more fragile banking business also is a more profitable banking business. In sharp contrast, indexation of cross-sectional local house price risk is highly effective at reducing mortgage defaults and financial fragility. It increases welfare for borrowers and depositors, while reducing it for intermediaries, as mortgage banking becomes safer but less profitable.

Recent proposals have emphasized indexation of interest payments but not principal, and envision only downward payment adjustments. We find that implementing such contracts would result in modest aggregate welfare gains, with borrowers and depositors gaining and intermediaries losing. The switch would lead to a substantial reduction in mortgage defaults, but a lower overall level of mortgage credit. Our results show that mortgage indexation in a world where intermediaries have limited liability can have important general equilibrium effects and must be designed carefully.



Table 2: Results: Main Indexation Experiments

	No Index	Aggregate	Local	Regional
<b>Borrower</b>				
1. Housing Capital	0.456	0.456	0.462	0.463
2. Refi rate	3.84%	3.81%	3.77%	3.74%
3. Default rate	0.95%	0.92%	0.49%	0.47%
4. Household leverage	64.41%	64.30%	65.79%	65.80%
5. Mortgage debt to income	259.59%	252.53%	274.88%	267.74%
6. Loss-given-default rate	38.61%	37.54%	38.03%	37.21%
7. Loss Rate	0.40%	0.39%	0.20%	0.20%
<b>Intermediary</b>				
8. Bank equity ratio	7.09%	7.33%	7.13%	7.25%
9. Bank default rate	0.33%	0.84%	0.22%	0.50%
10. DWL of bank defaults	0.07%	0.16%	0.05%	0.10%
11. Deposits	2.454	2.381	2.599	2.526
<b>Prices</b>				
12. House Price	8.842	8.595	9.042	8.784
13. Risk-free rate	0.71%	0.66%	0.74%	0.75%
14. Mortgage Rate	1.46%	1.54%	1.30%	1.35%
15. Credit spread	0.75%	0.87%	0.56%	0.60%
16. Mortgage Expec. Excess Ret	0.34%	0.49%	0.35%	0.40%
<b>Welfare</b>				
17. Aggregate welfare	0.821	+0.17%	+0.06%	+0.32%
18. Value function, B	0.379	-0.57%	+0.43%	+0.27%
19. Value function, D	0.374	-0.07%	+0.07%	+0.47%
20. Value function, I	0.068	+5.66%	-2.11%	-0.21%
<b>Consumption and Risk-sharing</b>				
21. Consumption, B	0.359	-0.3%	+0.3%	+0.1%
22. Consumption, D	0.372	-0.6%	+0.1%	+0.3%
23. Consumption, I	0.068	+6.1%	-2.9%	-0.4%
24. Consumption gr vol, B	0.42%	+351.3%	+15.9%	+189.0%
25. Consumption gr vol, D	1.11%	-10.4%	-26.5%	-15.4%
26. Consumption gr vol, I	4.47%	+392.9%	-54.1%	+282.5%
27. Wealth gr vol, I	0.035	+1366.8%	-1.8%	+679.3%
28. log (MU B / MU D) vol	0.025	-4.6%	-10.4%	-21.5%
29. log (MU B / MU I) vol	0.061	+145.7%	-36.8%	+101.8%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with full indexation of mortgage payments to aggregate house prices (second column), a model with indexation to relative local prices (third column), and a model with both aggregate and local indexation (fourth column). Rows 17-29 calculate percentage differences relative to the benchmark model in columns 2-4. All flow variables are quarterly.

Table 3: Results: Alternative Indexation Schemes

	No Index	Regional	Reg-IO	Reg-PO	Reg-Asym	Asym-IO
<b>Borrower</b>						
1. Housing Capital	0.456	0.463	0.458	0.463	0.468	0.461
2. Refi rate	3.84%	3.74%	3.84%	3.76%	4.42%	3.56%
3. Default rate	0.95%	0.47%	0.80%	0.49%	0.12%	0.55%
4. Household leverage	64.41%	65.80%	65.09%	65.63%	58.35%	62.85%
5. Mortgage debt to income	259.59%	267.74%	261.60%	270.80%	231.85%	260.24%
6. Loss-given-default rate	38.61%	37.21%	39.98%	39.67%	33.83%	30.44%
7. Loss Rate	0.40%	0.20%	0.32%	0.20%	0.91%	0.35%
<b>Intermediary</b>						
8. Bank equity ratio	7.09%	7.25%	7.15%	7.16%	7.33%	6.92%
9. Bank default rate	0.33%	0.50%	0.30%	0.31%	0.94%	0.34%
10. DWL of bank defaults	0.07%	0.10%	0.06%	0.07%	0.16%	0.07%
11. Deposits	2.454	2.526	2.484	2.553	2.196	2.373
<b>Prices</b>						
12. House Price	8.842	8.784	8.806	8.900	8.488	8.663
13. Risk-free rate	0.71%	0.75%	0.74%	0.74%	0.75%	0.77%
14. Mortgage Rate	1.46%	1.35%	1.41%	1.32%	2.37%	1.56%
15. Credit spread	0.75%	0.60%	0.70%	0.58%	1.62%	0.79%
16. Mortgage Expec. Excess Ret	0.34%	0.40%	0.35%	0.38%	0.49%	0.35%
<b>Welfare</b>						
17. Aggregate welfare	0.821	+0.32%	+0.20%	+0.18%	+0.73%	+0.25%
18. Value function, B	0.379	+0.27%	+0.30%	+0.33%	+1.85%	+0.53%
19. Value function, D	0.374	+0.47%	+0.25%	+0.21%	+0.07%	+0.37%
20. Value function, I	0.068	-0.21%	-0.61%	-0.75%	-1.91%	-2.02%
<b>Consumption and Risk-sharing</b>						
21. Consumption, B	0.359	+0.1%	+0.1%	+0.3%	+1.9%	+0.5%
22. Consumption, D	0.372	+0.3%	+0.3%	+0.2%	+0.0%	+0.4%
23. Consumption, I	0.068	-0.4%	-1.1%	-1.7%	-1.6%	-2.9%
24. Consumption gr vol, B	0.42%	+189.0%	-14.4%	+14.0%	+393.6%	+0.5%
25. Consumption gr vol, D	1.11%	-15.4%	-6.6%	-14.6%	-25.2%	-14.2%
26. Consumption gr vol, I	4.47%	+282.5%	+2.1%	+130.8%	+332.9%	-18.7%
27. Wealth gr vol, I	0.035	+679.3%	-2.6%	+286.4%	+1435.6%	+12.2%
28. log (MU B / MU D) vol	0.025	-21.5%	-5.8%	-33.7%	+11.9%	-10.2%
29. log (MU B / MU I) vol	0.061	+101.8%	-7.7%	+51.0%	+104.2%	-20.5%

The table reports averages from a long simulation (10,000 periods) of the benchmark model (first column), a model with regional indexation (second column), a model with regional interest indexation only (third column), a model with regional principal indexation only (fourth column), a model with regional asymmetric indexation (fifth column), and a model with regional asymmetric interest indexation only (sixth column). Rows 17-29 calculate percentage differences relative to the benchmark model in columns 2-6. All flow variables are quarterly.

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## A Model Derivations

### A.1 Derivation of Bank FOCs

First, starting from the value function in (24), we can define a value function net of the idiosyncratic profit shock

$$V^I(W_t^I, \mathcal{S}_t^I) = V_{ND}^I(W_t^I, \mathcal{S}_t^I) + \epsilon_t^I$$

such that we can equivalently write the optimization problem of the non-defaulting bank after the default decision as

$$V^I(W_t^I, \mathcal{S}_t^I) = \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I} W_t^I - J_t^I + \mathbb{E}_t \left[ \Lambda_{t,t+1}^I \max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0 \right\} \right], \quad (33)$$

subject to the same set of constraints as the original problem. We can now take the expectation with respect to  $\epsilon_t^I$  of the term in the expectation operator

$$\begin{aligned} & \mathbb{E}_\epsilon \left[ \max \left\{ V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I, 0 \right\} \right] \\ &= \text{Prob}_\epsilon \left( \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \mathbb{E}_\epsilon \left[ V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^I \mid \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right] \\ &= F_\epsilon^I \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right), \end{aligned} \quad (34)$$

with  $\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[ \epsilon_{t+1}^I \mid \epsilon_{t+1}^I < V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right]$  as in the main text. Inserting (34) into (33) gives the value function in (26) in the main text.

To derive the first-order conditions for the bank problem, we formulate the Lagrangian

$$\begin{aligned} \mathcal{L}^I(W_t^I, \mathcal{S}_t^I) &= \max_{L_t^*, \tilde{M}_t^I, \tilde{A}_t^I, B_{t+1}^I, \lambda_t^I} W_t^I - J_t^I + \mathbb{E}_t \left[ \Lambda_{t,t+1}^I F_\epsilon^I \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) \right) \left( V^I(W_{t+1}^I, \mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right] \\ &\quad + \lambda_t^I \left( \phi^I \left( q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - B_{t+1}^I \right) \right), \end{aligned} \quad (35)$$

and further conjecture that

$$V^I(W_t^I, \mathcal{S}_t^I) = W_t^I + \mathcal{C}(\mathcal{S}_t^I), \quad (36)$$

where  $\mathcal{C}(\mathcal{S}_t^I)$  is a function of the aggregate state variables but not bank net worth.

Before differentiating (35) to obtain first-order conditions, note that the derivative of the term in the expectation operator with respect to future wealth, after substituting in

this guess, is

$$\begin{aligned}
& \frac{\partial}{\partial W_{t+1}^I} F_\epsilon^I \left( W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left( W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \\
&= \frac{\partial}{\partial W_{t+1}^I} \left[ F_\epsilon^I \left( W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left( W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) - \int_{-\infty}^{W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I)} \epsilon f_\epsilon^I(\epsilon) d\epsilon \right] \\
&= F_\epsilon^I \left( W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right).
\end{aligned}$$

Using this result, and differentiating with respect to  $L_t^*$ ,  $\tilde{M}_t^I$ ,  $\tilde{A}_t^I$ ,  $B_{t+1}^I$ , and  $\lambda_t^I$  respectively, gives the first-order conditions

$$1 = q_t^M + r_t^* q_t^A, \quad (37)$$

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1} \zeta_{p,t+1} \left[ X_{t+1} + Z_{M,t+1} \left( (1-\delta) + \delta Z_{R,t+1} + \delta(1-Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}, \quad (38)$$

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1} \zeta_{p,t+1} \left[ Z_{A,t+1} \left( 1 + \delta(1-Z_{R,t+1}) q_{A,t+1}^A \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}, \quad (39)$$

$$q_t^f = \mathbb{E}_t \left[ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1} \right] + \lambda_t^I, \quad (40)$$

and the usual complementary slackness condition for  $\lambda_t^I$ . Recalling the definition of  $J_t^I$  as

$$J_t^I = (1 - r_t^* q_t^A - q_t^M) L_t^* + q_t^A \tilde{A}_t^I + q_t^M \tilde{M}_t^I - q_t^f B_{t+1}^I,$$

we note that the term in front of  $L_t^*$  is zero due to FOC (37), and we can substitute out prices  $q_t^M$ ,  $q_t^A$ , and  $q_t^f$  from FOCs (38)-(40), both in  $J_t^I$  and in the constraint term in (35).

Further inserting our guess from (36) on the left-hand side of (35), and canceling and collecting terms, we get

$$\mathcal{C}(\mathcal{S}_t^I) = \mathbb{E}_t \left[ \Lambda_{t,t+1}^I F_\epsilon^I \left( W_{t+1}^I + \mathcal{C}(\mathcal{S}_{t+1}^I) \right) \left( \mathcal{C}(\mathcal{S}_{t+1}^I) - \epsilon_{t+1}^{I,-} \right) \right], \quad (41)$$

which confirms the conjecture.  $\mathcal{C}(\mathcal{S}_t^I)$  is the recursively defined value of the bankruptcy option to the bank. Note that without the option to default, one gets

$$\epsilon_{t+1}^{I,-} = \mathbb{E}_\epsilon \left[ \epsilon_{t+1}^I \right] = 0.$$

Then the equation in (41) implies that  $\mathcal{C}(\mathcal{S}_t^I) = 0$  and thus  $V^I(W_t^I, \mathcal{S}_t^I) = W_t^I$ . However, if the bank has the option to default, its value generally exceeds its financial wealth  $W_t^I$  by the bankruptcy option value  $\mathcal{C}(\mathcal{S}_t^I)$ .

## A.2 Aggregation of Intermediary Problem

Before aggregating across loans, we must treat the distribution over  $m_t(r)$ , the start-of-period balance of a loan with interest rate  $r$ , as a state variable. In addition, the intermediary can freely choose her end-of-period holdings of these loans  $\tilde{m}_t(r)$  by trading in the secondary market at price  $q^m(r)$ . In this case, the intermediary's problem is to choose nondurable consumption  $C_t^I$ , new debt issuance  $L_t^*$ , new deposits  $B_{t+1}^I$ , new REO investment  $I_t^{REO}$ , and end-of-period loan holdings  $\tilde{m}_t(r)$  to maximize (2) subject to the budget constraint

$$\begin{aligned}
C_t^I = & \underbrace{(1 - \tau)Y_t^I}_{\text{disp. income}} + \underbrace{\int \left[ X_t + Z_{A,t}r + Z_{M,t} \left( (1 - \delta) + \delta Z_{R,t} \right) \right] m_t(r) dr}_{\text{payments on existing debt}} - \underbrace{(1 - q_t^m(r_t^*))L_t^*}_{\text{net new debt}} \\
& + \underbrace{q_t^f B_{t+1}^I - \bar{\pi}^{-1} B_t^I}_{\text{net deposits}} - \underbrace{\int q_t^m(r) \left[ \tilde{m}_t(r) - \delta(1 - Z_{R,t})Z_{M,t}m_t(r) \right] dr}_{\text{secondary market trades}} \\
& + \underbrace{\left[ \rho_t + (S^{REO} - \nu^{REO}) p_t \right] K_t^{REO}}_{\text{REO income}} - \underbrace{p_t^{REO} \left[ I_t^{REO} - X_t A_t^I \right]}_{\text{REO investment}}
\end{aligned} \tag{42}$$

and the leverage constraint

$$q_t^f B_t^* \leq \phi^M \int q_t^m(r) \tilde{m}_t(r) dr + \phi^{REO} p_t^{REO} \tilde{K}_t^{REO}$$

with the laws of motion

$$\begin{aligned}
m_{t+1}(r) &= \bar{\pi}^{-1} \zeta_{p,t+1} \tilde{m}_t(r) \\
K_{t+1}^{REO} &= (1 - S^{REO}) K_t^{REO} + (1 - Z_{K,t}) K_t^B
\end{aligned}$$

and where the recovery rate  $X_t$  is defined as before. From the optimality condition for end-of-period holdings for loans with a given interest rate  $\tilde{m}_t(r)$ , we obtain

$$q_t^m(r) = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I \bar{\pi}^{-1} \zeta_{p,t+1} \left[ X_{t+1} + Z_{A,t+1}r + Z_{M,t+1} \left( (1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1})q_{t+1}^m(r) \right) \right] \right\}}{1 - \lambda_t^I \phi^M}$$



where  $\lambda_t^I$  is the multiplier on the intermediary's leverage constraint. To obtain aggregation, we can split  $q_t(r)$  into an interest-only strip with value  $q_t^M$  and a principal-only strip with value  $q_t^A$ , so that

$$q_t^m(r) = r q_t^A + q_t^M.$$

Substituting into the equilibrium condition for  $q_t^m(r)$  verifies the conjecture and yields

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I Y_{t+1}^M \left[ X_{t+1} + Z_{M,t+1} \left( (1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{1 - \lambda_t^I \phi^M}$$

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I Y_{t+1}^M Z_{A,t+1} \left[ 1 + \delta(1 - Z_{R,t+1}) q_{t+1}^A \right] \right\}}{1 - \lambda_t^I \phi^M}.$$

Importantly, due to our assumption on the prepayment behavior of borrowers (ensuring a constant  $Z_{R,t}$  across the  $r$  distribution), the prices  $q_t^A$  and  $q_t^M$  are independent of  $r$ . Substituting into the budget constraint, and applying the identities

$$M_t^I = \int m_t(r) dr$$

$$A_t^I = \int r m_t(r) dr$$

now yields the aggregated budget constraint (15) and leverage constraint (21).

## B Model Solution

**Borrower Optimality.** The optimality condition for new mortgage debt,

$$1 = \Omega_{M,t} + r_t^* \Omega_{A,t} + \lambda_t^{LTV},$$

equalizes the benefit of taking on additional debt — \$1 today — to the cost of carrying more debt in the future, both in terms of carrying more principal ( $\Omega_{M,t}$ ) and higher interest payments ( $\Omega_{A,t}$ ), plus the shadow cost of tightening the LTV constraint. The marginal continuation costs are defined recursively:

$$\Omega_{M,t} = \mathbb{E}_t \left\{ \Lambda_{t+1}^B \bar{\pi}^{-1} \zeta_{p,t+1} Z_{M,t+1} \left[ (1 - \delta) + \delta Z_{R,t+1} + \delta(1 - Z_{R,t+1}) \Omega_{M,t+1} \right] \right\}$$

$$\Omega_{A,t} = \mathbb{E}_t \left\{ \Lambda_{t+1}^B \bar{\pi}^{-1} \zeta_{p,t+1} Z_{A,t+1} \left[ (1 - \tau) + \delta(1 - Z_{R,t+1}) \Omega_{A,t+1} \right] \right\}$$

where an extra unit of principal requires a payment of  $(1 - \delta)$  in the case of non-default, plus payment of the face value of prepaid debt, plus the continuation cost of non-prepaid debt. An extra promised payment requires a tax-deductible payment on non-defaulted debt plus the continuation cost if the debt is not prepaid.

The optimality condition for housing services consumption sets the rental rate to be the marginal rate of substitution between housing services and nondurables:

$$\rho_t = \frac{u_{H,t}}{u_{C,t}} = \left( \frac{\tilde{\zeta}_t}{1 - \tilde{\zeta}_t} \right) \left( \frac{C_t^B}{H_t^B} \right)$$

The borrower's optimality condition for new housing capital is:

$$p_t = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^B \left[ \rho_{t+1} + Z_{K,t+1} p_{t+1} \left( 1 - \nu^K - (1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K \right) \right] \right\}}{1 - \lambda_t^{LTV} \phi^K}.$$

The numerator represents the present value of holding an extra unit of housing next period: the rental service flow, plus the continuation value of the housing if the borrower chooses not to default, net of the maintenance cost. The continuation value needs to be adjusted by  $(1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K$  because if the borrower does not choose to refinance, which occurs with probability  $1 - Z_{R,t+1}$ , then she does not use the unit of housing to collateralize a new loan, and therefore does not receive the collateral benefit.

The optimal refinancing rate is:

$$Z_{R,t} = \Gamma_\kappa \left\{ \underbrace{\left( 1 - \Omega_{M,t} - \bar{r}_t \Omega_{A,t} \right) \left( 1 - \frac{\delta Z_{M,t} M_t}{Z_{N,t} M_t^*} \right)}_{\text{equity extraction incentive}} + \underbrace{\Omega_{A,t} (\bar{r}_t - r_t^*)}_{\text{interest rate incentive}} - \underbrace{p_t \lambda_t^{LTV} \phi^K \left( \frac{Z_{N,t} K_t^* - Z_{K,t} K_t^B}{Z_{N,t} M_t^*} \right)}_{\text{collateral expense}} \right\} \quad (43)$$

where  $\bar{r}_t = A_t^B / M_t^B$  is the average interest rate on existing debt. The “equity extraction incentive” term represents the net gain from obtaining additional debt at the *existing* interest rate, while “interest rate incentive” term represents the gain from moving from the existing to new interest rate. The stronger these incentives, the higher the refinancing rate. The “collateral expense” term arises because housing trades at a premium relative to the present value of its housing service flow due to its collateral value. If the borrower

intends to obtain new debt by buying more housing collateral, the cost of paying this premium must be taken into account.

The optimality condition for the default rate pins down the default threshold  $\bar{\omega}_t^U$  as a function of the aggregate state, as well as the value of the local component ( $\omega_{i,t}^L$ ):

$$\bar{\omega}_t^U = \frac{(\omega_{i,t}^L)^{\iota_\omega} (Q_{A,t}A_t + Q_{M,t}M_t)}{\omega_{i,t}^L Q_{K,t}K_t^B} \quad (44)$$

where  $Q_{A,t}$  and  $Q_{M,t}$  are the marginal benefits of discharging interest payments and principal, respectively, and  $Q_{K,t}$  is the marginal continuation value of housing, defined by

$$Q_{A,t} = \underbrace{(1 - \tau)}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{A,t}}_{\text{continuation cost}} \quad (45)$$

$$Q_{M,t} = \underbrace{(\delta Z_{R,t} + (1 - \delta))}_{\text{current payment}} + \underbrace{\delta(1 - Z_{R,t})\Omega_{M,t}}_{\text{continuation cost}} \quad (46)$$

$$Q_{K,t} = \left[ \underbrace{Z_{R,t}}_{\text{refi case}} + \underbrace{(1 - Z_{R,t}) \left(1 - \lambda_t^{LTV} \phi^K\right)}_{\text{no refi case}} - \underbrace{v^K}_{\text{maint.}} \right] p_t. \quad (47)$$

Equation (44) relates the benefit of defaulting on debt, which is eliminating both the current payment and continuation cost, potentially indexed by  $\omega_{i,t}^L$ , against the cost of losing a marginal unit of housing, which has been scaled by both  $\omega_{i,t}^L$  and  $\omega_{i,t}^U$ . The marginal value of housing  $Q_{K,t}$  is equal to the full market price  $p_t$  net of maintenance if used to collateralize a new loan (i.e., if the borrower refinances) but is worth less if the borrower does not refinance next period due to the loss of collateral services.

**Intermediary Optimality.** The optimality condition for new debt  $L^*$  is:

$$1 = q_t^M + r_t^* q_t^A,$$

which balances the cost of issuing new debt, \$1 today, against the value of the loan obtained, 1 unit of PO strip plus  $r_t^*$  units of the IO strip. The condition implies that the first term in (20) is zero.

The optimality condition for deposits is:

$$q_t^f = \mathbb{E}_t \left[ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1} \right] + \lambda_t^I$$

where  $\lambda_t^I$  is the multiplier on the intermediary's leverage constraint (21). The default option, represented by the  $F_{\epsilon,t+1}^I$  term in the expectation, drives a wedge between the valuation of risk-free debt by intermediary households,  $\mathbb{E}_t \left[ \Lambda_{t+1}^I \bar{\pi}^{-1} \right]$ , and that of banks.

The optimality conditions for PO and IO strip holdings pin down their prices:

$$q_t^M = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1} \zeta_{p,t+1} \left[ X_{t+1} + Z_{M,t+1} \left( (1 - \delta) + \delta Z_{R,t+1} + \delta (1 - Z_{R,t+1}) q_{t+1}^M \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}$$

$$q_t^A = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^I F_{\epsilon,t+1}^I \bar{\pi}^{-1} \zeta_{p,t+1} \left[ Z_{A,t+1} \left( 1 + \delta (1 - Z_{R,t+1}) q_{t+1}^A \right) \right] \right\}}{(1 - \phi^I \lambda_t^I)}.$$

Both securities issue cash flows that are nominal (discounted by inflation) and indexed to house prices (discounted by  $\zeta_{p,t+1}$ ). Both securities can also be used to collateralize deposits, leading to the collateral premia in the denominators. The IO strip's next-period payoff is equal to \$1 for loans that do not default, with a continuation value of  $q_{t+1}^A$  for loans that do not prepay or mature. The PO strip's next-period payoff is the recovery value for defaulting debt  $X_{t+1}$  plus the payoff from loans that do not default: the principal payment  $1 - \delta$ , plus the face value of prepaying debt, plus the continuation value  $q_{t+1}^M$  for loans that do not mature or prepay.

The optimality condition for REO housing is:

$$p_t^{REO} = \mathbb{E}_t \left\{ \Lambda_{t+1}^I \left[ \rho_{t+1} - v^{REO} p_{t+1} + S^{REO} p_{t+1} + (1 - S^{REO}) p_{t+1}^{REO} \right] \right\}.$$

The right-hand side is the present discounted value of holding a unit of REO housing next period. This term is in turn made up of the rent charged to borrowers, the maintenance cost, and the value of the housing next period, both the portion sold back to the borrowers, and the portion kept in the REO state.

**Depositor Optimality.** The depositor's sole optimality condition for deposits, which are nominal contracts, ensures that the depositor's Euler equation is at an interior solution:

$$q_t^f = \mathbb{E}_t \left[ \Lambda_{t+1}^D \bar{\pi}^{-1} \right].$$

## C Online Appendix: Model Extensions

### C.1 IO/PO Indexation

The default thresholds with interest-only (IO) and principal-only (PO) mortgage payment indexation are given by:

$$\begin{aligned} \text{Interest Only : } \quad \bar{\omega}_t^U &= \frac{Q_{A,t}A_t + (\omega_{i,t}^L)^{\iota_\omega} Q_{M,t}M_t}{\omega_{i,t}^L Q_{K,t} p_t K_t^B} \\ \text{Principal Only : } \quad \bar{\omega}_t^U &= \frac{(\omega_{i,t}^L)^{\iota_\omega} Q_{A,t}A_t + Q_{M,t}M_t}{\omega_{i,t}^L Q_{K,t} p_t K_t^B} \end{aligned}$$

which are identical to (44) with the exception that only one component or the other in the numerator is indexed, but not both. Additionally,  $Z_{M,t}$  is replaced by  $Z_{N,t}$  and  $\zeta_{p,t}$  is replaced by 1 when scaling principal balances in the interest-only case. Symmetrically,  $Z_{A,t}$  is replaced by  $Z_{N,t}$  and  $\zeta_{p,t}$  is replaced by 1 when scaling interest payments  $A_t$  in the interest-only case.

### C.2 Asymmetric Indexation

Under asymmetric indexation, we now need to switch from i.i.d. to persistent processes for the  $\omega_{i,t}$  terms. When default is symmetric, the influence of local indexation occurs only through default, which depends on overall cross-sectional dispersion, not the specific innovations themselves. However, under asymmetric indexation the average loss that the bank takes depends on the volatility of the innovation, not on the overall dispersion. To accommodate this, we now allow the  $\omega_{i,t}$  components to follow independent AR(1) processes in logs

$$\log \omega_{i,t}^L = (1 - \rho_\omega) \mu_{\omega,t}^L + \rho_\omega \log \omega_{i,t-1}^L + \left( \frac{\sigma_{\omega,t}}{\sqrt{1 - \rho_\omega^2}} \right) e_{i,t}^L, \quad e_{i,t}^L \sim N(0, \alpha) \quad (48)$$

$$\log \omega_{i,t}^U = (1 - \rho_\omega) \mu_{\omega,t}^U + \rho_\omega \log \omega_{i,t-1}^U + \left( \frac{\sigma_{\omega,t}}{\sqrt{1 - \rho_\omega^2}} \right) e_{i,t}^U, \quad e_{i,t}^U \sim N(0, 1 - \alpha) \quad (49)$$

where the shocks  $e_{i,t}^L$  and  $e_{i,t}^U$  are uncorrelated and account for  $\alpha$  and  $1 - \alpha$  of the cross-sectional variance of  $\omega_{i,t}$ , respectively. The means  $\mu_{\omega,t}^L$  and  $\mu_{\omega,t}^U$  are set to

$$\mu_{\omega,t}^L = \frac{1}{2} \frac{\alpha \sigma_{\omega,t}^2}{1 - \rho_{\omega}}, \quad \mu_{\omega,t}^U = \frac{1}{2} \frac{(1 - \alpha) \sigma_{\omega,t}^2}{1 - \rho_{\omega}}$$

so that the cross-sectional average of each component  $\omega_{i,t}^L$  and  $\omega_{i,t}^U$  is unity at each date. To ensure that the cross-sectional variance is always equal to  $\sigma_{\omega,t}^2$ , we assume that at the start of a recession, (48) and (49) receive special additional innovations  $\tilde{e}_{i,t}^L$  and  $\tilde{e}_{i,t}^U$ , where

$$\tilde{e}_{i,t}^L \sim N \left( 0, \alpha \frac{\bar{\sigma}_{\omega,1}^2 - \bar{\sigma}_{\omega,0}^2}{1 - \rho_{\omega}^2} \right) \quad \tilde{e}_{i,t}^U \sim N \left( 0, (1 - \alpha) \frac{\bar{\sigma}_{\omega,1}^2 - \bar{\sigma}_{\omega,0}^2}{1 - \rho_{\omega}^2} \right).$$

These shocks are reversed at the end of the financial recession, and are re-drawn in each financial recession.

Under asymmetric indexation, equations (6) and (7) become:

$$\zeta_{p,t} = \min \left\{ \left( \frac{p_t}{p_{t-1}} \right)^{l_p}, \bar{\zeta}_p \right\}$$

$$\zeta_{\omega,t}(\omega_{i,t-1}^L, \omega_{i,t}^L) = \min \left\{ \left( \frac{\omega_{i,t}^L}{\omega_{i,t-1}^L} \right)^{l_{\omega}}, \bar{\zeta}_{\omega} \right\}$$

As a result, the default threshold is now defined by

$$\bar{\omega}_t^U = \frac{\min \left\{ \omega_{i,t}^L, \bar{\zeta}_{\omega} \omega_{i,t-1}^L \right\} (Q_{A,t} A_t + Q_{M,t} M_t)}{\omega_{i,t}^L Q_{K,t} K_t^B}$$

which, all else equal, weakly lowers the default threshold since local indexation terms become more generous when local house price growth is high. For our asymmetric indexation experiments, we set  $\bar{\zeta}_p = \bar{\zeta}_{\omega} = 1$ , implying that mortgages are never indexed upward, but only downward.

We calibrate the parameter  $\rho_{\omega}$  to 0.977 based on FHFA MSA-level house price indices, as discussed in Section 3.

For the quantity of housing retained by the borrower, the only update needed is an extra integral to account for the dependence of  $\bar{\omega}_t^U$  on both the lagged value of the local

component and its innovation:

$$Z_{K,t} = \int \int \left( \int_{\omega_{i,t}^U > \bar{\omega}_t^U} \omega_{i,t}^U d\Gamma_{\omega,t}^U \right) \omega_{i,t}^L d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L.$$

Finally, the quantity of debt retained by the borrowers needs to be updated both for this change in the dependence of  $\bar{\omega}_t^U$ , as well as the cap on how much debt can be upwardly indexed:

$$Z_{M,t} = Z_{A,t} = \int \int \underbrace{\left( 1 - \Gamma_{\omega,t}^U \left( \bar{\omega}_t^U(\omega_{i,t-1}^L, e_{i,t}^L) \right) \right)}_{\text{remove defaulters}} \underbrace{(\omega_{i,t}^L)^{l\omega}}_{\text{indexation}} d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L$$

In the case of interest-only asymmetric indexation,  $Z_{K,t}$ ,  $Z_{A,t}$ , and  $Z_{N,t}$  are computed as above,  $Z_{M,t} = Z_{N,t}$ , and  $\zeta_{p,t}$  is replaced by 1 in the transition equation for principal balances  $M_t^B$ .

For all asymmetric cases, to maintain tractability, we account for updates to the average level of debt indexation through adjustments to  $Z_{A,t}$  and  $Z_{M,t}$  due to this asymmetry, but maintain the assumption — now only approximate — that the unconditional distribution of indexation still follows the same log-normal distribution around this mean.

The asymmetric IO-indexation case features the default threshold

$$\bar{\omega}_t^U = \frac{\min \left\{ \omega_{i,t}^L, \bar{\zeta}_\omega \omega_{i,t-1}^L \right\} Q_{A,t} A_t + Q_{M,t} M_t}{\omega_{i,t}^L Q_{K,t} p_t K_t^B},$$

which lies in between the Reg-IO and Reg-Asym models.

### C.3 Transition Path Results

Table 4 shows the change in variables in the first period of transition on the path between the “No Index” steady state, and the steady state of an alternative model. This is particularly useful since the value functions will measure the total welfare change including the entire transition path to the new steady state.

### C.4 Liquidity Defaults

This section considers a model extension where defaults are driven by both liquidity concerns (the need to stop making mortgage payments) and strategic motives.

Table 4: Transition Path Impacts (Alternative Indexation Schemes)

	No Index	Regional	Reg-IO	Reg-PO	Reg-Asym	Asym-IO
<b>Welfare and Consumption</b>						
Welfare	0.821	+0.61%	+0.36%	+0.51%	+0.90%	+0.28%
Value function, B	0.379	+0.68%	+0.61%	+0.83%	+1.76%	+0.36%
Value function, D	0.374	+0.54%	+0.34%	+0.28%	+0.11%	+0.47%
Value function, I	0.068	+0.53%	-0.95%	-0.03%	+0.51%	-1.25%
Consumption, B	0.359	+0.50%	+0.78%	+1.11%	-1.00%	-0.18%
Consumption, D	0.372	+0.82%	+1.49%	+0.32%	+0.47%	+2.42%
Consumption, I	0.068	+4.63%	-1.09%	+3.00%	+18.26%	+0.35%
<b>Housing and Mortgage Market</b>						
Deposits	2.454	+5.98%	+5.84%	+6.52%	-8.34%	+3.79%
House Price	8.842	+2.30%	+2.58%	+3.55%	-2.11%	+0.73%
Mortgage debt to income	2.596	+4.76%	+4.76%	+4.76%	+4.76%	+4.76%
Mortgage rate	1.46	-0.04%	-0.05%	-0.07%	+0.80%	+0.06%
Refi Rate	3.84	-0.00%	+0.07%	+0.10%	-0.82%	-0.15%
Loss Rate	0.40	-0.33%	-0.24%	-0.33%	+0.42%	-0.11%
Bank default rate	0.33	-0.24%	-0.19%	-0.21%	-0.29%	-0.20%

The table reports the initial change following a surprise switch from the baseline mortgage contract (“no index”) to an alternative contract. Each transition path is computed from a random starting point simulated from the stationary distribution of the benchmark model. All flow variables are quarterly.

**Model** To allow for liquidity defaults, assume that fraction  $\theta$  of borrowers are hit by liquidity shocks in each period. After being hit with the shock, borrowers have the choice of whether to sell the house or to default. We assume that borrowers hit with the liquidity shock only default if they are under water.

Define the  $\omega_{i,t}^U$  threshold for default conditional on receiving a liquidity shock as  $\bar{\omega}_{i,t}^{U,Liq}$ . The choice between selling the house and defaulting implies:

$$\bar{\omega}_{i,t}^{U,Liq} = \frac{\mathcal{I}_M \left\{ \min \left( \omega_{i,t}^L, \bar{\zeta} \omega_{i,t-1}^L \right) \right\} M_t}{\omega_{i,t}^L (1 - v^K) p_t K_t},$$

where  $\mathcal{I}_M\{x\}$  is equal to  $x$  if the principal is indexed, and is equal to 1 if the principal is not indexed.

Conditional on receiving a liquidity shock, we have (using the general  $\omega_{i,t}$  notation



from Section C.2):

$$\begin{aligned}
Z_{D,t}^{Liq} &= \int \int \Gamma_{\omega,t}^U \left( \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
Z_{N,t}^{Liq} &= \int \int \Gamma_{\omega,t}^U \left( 1 - \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
Z_{K,t}^{Liq} &= \int \int \int_{\bar{\omega}_{i,t}^{U,Liq}(\omega^L)} \omega_{i,t}^U d\Gamma_{\omega,t}^U d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
Z_{M,t}^{Liq} &= \int \int \Gamma_{\omega,t}^U \left( 1 - \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) \mathcal{I}_M(\omega^L) d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
Z_{A,t}^{Liq} &= \int \int \Gamma_{\omega,t}^U \left( 1 - \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) \mathcal{I}_A(\omega^L) d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L.
\end{aligned}$$

For strategic default, we introduce an extra cost to the borrower of losing his or her home, equal to  $\eta^B$  of the value of the home. This allows us to capture the observation that borrowers do not tend to strategically default until they are well under water. In this case, the strategic default threshold becomes:

$$\bar{\omega}_{i,t}^{U,Str} = \frac{\mathcal{I}_M \left\{ \min \left( \omega_{i,t}^L, \bar{\zeta}_\omega \omega_{i,t-1}^L \right) \right\} Q_{M,t} M_t^B + \mathcal{I}_A \left\{ \min \left( \omega_{i,t}^L, \bar{\zeta}_\omega \omega_{i,t-1}^L \right) \right\} Q_{A,t} A_t^B}{(1 + \eta^B) \omega_{i,t}^L Q_{K,t} K_t^B}$$

where the  $Q$  terms are defined as above. Given this threshold, the corresponding  $Z^{Str}$  values can be computed by replacing  $Liq$  with  $Str$  above.

Total default rates are the weighted average of liquidity and strategic default rates with weights  $\theta$  and  $1 - \theta$ . More generally, the  $Z$  variables can now be computed as follows:

$$Z_{N,t} = \theta Z_{N,t}^{Liq} + (1 - \theta) Z_{N,t}^{Str}$$

which holds assuming that the liquidity default threshold is always strictly above the strategic default threshold, i.e., no households receiving liquidity shocks that do not liquidity default would choose to strategically default.

The borrower's budget constraint becomes:

$$\begin{aligned}
C_t^B = & \underbrace{(1 - \tau)Y_t^B}_{\text{disp. income}} + \underbrace{Z_{R,t} \left( Z_{N,t}M_t^* - \delta Z_{M,t}M_t^B \right)}_{\text{net new borrowing}} - \underbrace{(1 - \delta)Z_{M,t}M_t^B}_{\text{principal payment}} - \underbrace{(1 - \tau)Z_{A,t}A_t^B}_{\text{interest payment}} \\
& - \underbrace{p_t \left[ Z_{R,t}Z_{N,t}K_t^* + (v^K - Z_{R,t}) Z_{K,t}K_t^B \right]}_{\text{owned housing}} - \underbrace{\rho_t \left( H_t^B - K_t^B \right)}_{\text{rental housing}} \\
& - \underbrace{\left( \Psi(Z_{R,t}) - \bar{\Psi}_t \right) Z_{N,t}M_t^*}_{\text{net transaction costs}} - \underbrace{\eta^B (1 - Z_{K,t}) p_t K_t^B - \text{Rebate}_t}_{\text{foreclosure costs}} - \underbrace{T_t^B}_{\text{lump sum taxes}}.
\end{aligned} \tag{50}$$

We rebate the utility cost from foreclosure lump-sum.

Finally, we allow borrowers to internalize the effect of their housing and debt decisions on the probability of liquidity default. Since the liquidity default threshold is mechanical, unlike the strategic default threshold which is optimally chosen, the envelope theorem does not apply, and the response of the liquidity default probability will enter the borrower's optimality conditions.

To aid notation, define  $\Delta_{x,t}^M = \theta \partial Z_{M,t}^{Liq} / \partial x$  for a given variable  $x$  and superscript  $M$ , and define  $\Delta_{x,t}^{BC}$  to be the derivative of the budget constraint with respect to  $x_t$ . Then we have:

$$\begin{aligned}
\Delta_{x,t}^{BC} = & Z_{R,t} \left( \Delta_{x,t}^N M_t^* - \delta \Delta_{x,t}^M M_t^B \right) - (1 - \delta) \Delta_{x,t}^M M_t^B - (1 - \tau) \Delta_{x,t}^A A_t^B \\
& - p_t \left[ Z_{R,t} \Delta_{x,t}^N K_t^* + (v^K - Z_{R,t}) \Delta_{x,t}^K K_t^B \right] - \left( \Psi(Z_{R,t}) - \bar{\Psi}_t \right) \Delta_{x,t}^N M_t^* + \eta^B \Delta_{x,t}^K p_t K_t^B
\end{aligned}$$

where

$$\begin{aligned}
\frac{\partial Z_{D,t}^{Liq}}{\partial x} &= \int \int f_{\omega,t}^U \left( \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) \frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial x} d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
\frac{\partial Z_{N,t}^{Liq}}{\partial x} &= - \int \int f_{\omega,t}^U \left( \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) \frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial x} d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
\frac{\partial Z_{K,t}^{Liq}}{\partial x} &= - \int \int \bar{\omega}_{i,t}^{U,Liq} \frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial x} d\Gamma_{\omega,t}^U d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
\frac{\partial Z_{M,t}^{Liq}}{\partial x} &= - \int \int f_{\omega,t}^U \left( \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) \frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial x} \mathcal{I}_M(\omega^L) d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L \\
\frac{\partial Z_{A,t}^{Liq}}{\partial x} &= - \int \int f_{\omega,t}^U \left( \bar{\omega}_{i,t}^{U,Liq}(\omega^L) \right) \frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial x} \mathcal{I}_A(\omega^L) d\Gamma_{e,t}^L d\Gamma_{\omega,t-1}^L.
\end{aligned}$$

The derivatives of the threshold  $\bar{\omega}_{i,t}^{U,Liq}$  with respect to the state variables are

$$\begin{aligned}\frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial K_t^B} &= -\frac{\mathcal{I}_M \left\{ \min \left( \omega_{i,t}^L, \bar{\zeta} \omega_{i,t-1}^L \right) \right\} M_t}{\omega_{i,t}^L p_t K_t^2} \\ \frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial M_t^B} &= \frac{\mathcal{I}_M \left\{ \min \left( \omega_{i,t}^L, \bar{\zeta} \omega_{i,t-1}^L \right) \right\}}{\omega_{i,t}^L p_t K_t} \\ \frac{\partial \bar{\omega}_{i,t}^{U,Liq}}{\partial A_t^B} &= 0\end{aligned}$$

which captures the fact that only principal balance and not promised interest payments influence liquidity default.

The first-order condition for housing becomes:

$$p_t = \frac{\mathbb{E}_t \left\{ \Lambda_{t+1}^B \left[ \rho_t + \Delta_{K,t+1}^{BC} - (1 - Z_{K,t+1}) \eta^B p_{t+1} + Z_{K,t+1} p_{t+1} (1 - \nu^K - (1 - Z_{R,t+1}) \lambda_{t+1}^{LTV} \phi^K) \right] \right\}}{1 - \lambda_t^{LTV} \phi^K}.$$

The marginal continuation cost of debt becomes:

$$\Omega_{M,t} = \mathbb{E}_t \left\{ \Lambda_{t+1}^B \bar{\pi}^{-1} \zeta_{p,t+1} Z_{M,t+1} \left[ \Delta_{M,t+1}^{BC} + (1 - \delta) + \delta Z_{R,t+1} + \delta (1 - Z_{R,t+1}) \Omega_{M,t+1} \right] \right\}.$$

Since the liquidity default threshold does not depend directly on interest payments, there is no additional adjustment required for the continuation cost of interest payments,  $\Omega_{A,t}$ .

**Calibration** We calibrate  $\theta = .175$  and  $\eta_B = .04$  to (i) match the overall default rate to that in the data (and in the baseline model with only strategic defaults, which matches the data), and (ii) to generate a fraction of liquidity defaults of 55%. While it is difficult to find a precise target for the fraction of liquidity defaults, most economists agree that a majority of defaults in the financial crisis were of the liquidity default type (Foote, Gerardi, and Willen, 2008). Still, there was a non-trivial fraction of strategic defaults (Keys et al., 2012), and recent evidence from investors further bolsters the case for a substantial strategic default share (Foote, Loewenstein, Nosal, and Willen, 2018). The calibration captures the observation that some households do not strategically default until they are well under water (Guiso, Sapienza, and Zingales, 2013). All other parameters are kept at the values of the model in the main text.

**Results** Table 5 shows the results for prices, quantities, and welfare of the model that allows for liquidity defaults. For ease of comparison, the first column repeats the No Index case in the baseline model with only strategic defaults. There are two key findings. First, the No Index cases without and with liquidity default are very similar (columns 1 and 2). The only small difference is that house prices are slightly higher in the model with liquidity defaults, with a commensurate increase in mortgage balances and deposits. This is mostly due to borrowers internalizing that accumulating more housing reduces their probability of liquidity default, increasing the marginal value of housing — an alternative model in which borrowers ignore this internalization (not shown) delivers results almost indistinguishable from the baseline.

To understand this near-equivalence, recall that the key differences between liquidity and strategic default is that liquidity default depends only on principal balance, not on promised interest payments, and that the liquidity default threshold is mechanical, while the strategic default threshold optimally depends on the continuation costs  $Q_{A,t}$ ,  $Q_{M,t}$ ,  $Q_{K,t}$  defined in equations (45) - (47). First, as in the discussion on IO/PO indexation, ignoring the continuation costs of promised interest payments has only a small effect because changes in promised interest payments only matter until the next loan renewal, weakening their impact. Second, as a quantitative matter, the endogenous effects on the  $Q$  terms largely cancel out, meaning that the optimal strategic threshold co-moves closely with the mechanical liquidity threshold.

Turning to welfare, the headline welfare changes from Aggregate, Local, and Regional compared to No Index are quantitatively similar in the model with liquidity default, compared to the baseline model without liquidity defaults. Like in the model without liquidity default, the model with liquidity default obtains the largest welfare gain from regional asymmetric indexation. The winners and losers in each policy experiment are the same, and the magnitude of the gains and losses is similar.

## C.5 Rebating Housing Maintenance

In the baseline model, housing maintenance (residential investment) moves with house prices and affects resources available for consumption. Since house prices are endogenous, different indexation regimes lead to house prices, and therefore different amounts of resources available for consumption. As a result, indexation regimes that increase house prices will impose a welfare cost through an increase in housing maintenance expenses. As a robustness exercise, we change this assumption, and instead rebate housing

maintenance costs in lump-sum fashion in households' budget constraints. All parameters are the same as in the baseline model.

Table 6 shows the results for prices, quantities, and welfare of the model that rebates maintenance expenses. The first two columns compare the No Index cases for the baseline model and the model where maintenance is rebated. The model with maintenance rebates has higher house prices than the baseline, due to the fact that higher nondurable consumption increases the marginal utility of housing services. Higher house prices support larger amounts of mortgage debt-to-income, and a larger banking sector (deposits). Deposits are not only larger, but also more volatile. Thus, the model with rebating features increased depositor consumption growth volatility, since depositors need to accommodate greater fluctuations in deposit supply from banks as the banking sector contracts during crises and the expands again in the recovery. A more volatile risk-free rate is required to induce these greater depositor consumption swings. In particular, a sharper drop in the rate during financial crises is necessary in equilibrium to achieve a larger consumption spike for depositors as constrained banks cut the deposit supply. As a result, the average level of the risk-free rate is lower. Mortgage rates are slightly lower as well, largely due to the fall in the risk-free rate.

The welfare gains of indexation schemes (relative to the no indexation case) are similar in the model with maintenance rebates and the baseline model without rebates. The winners and losers of aggregate, local, and regional indexation are the same, and so are the magnitudes of the gains and losses. With symmetric local indexation, the borrowers gain more (+1.11% vs. +0.43%) and the intermediaries do not lose as much (-1.16% vs. -2.11%) with rebate maintenance, leading to a larger gain from local indexation (+0.43% vs. +0.06%). The same finding holds for the Regional case (+0.59% vs. +0.32% without rebates). The Reg-Asym model has a substantially smaller gain of +0.24% relative to No Index, compared to a gain of +0.73% in the baseline model without rebates. This shows that a large share of the original welfare gain from asymmetric regional indexation stemmed from lower maintenance costs, as this scheme substantially lowers house prices compared to the No Index case.

Table 5: Results: Liquidity Default

	Baseline	Model with Liquidity Defaults				
	No Index	No Index	Aggregate	Local	Regional	Reg-Asym
<b>Borrower</b>						
1. Housing Capital	0.456	0.455	0.455	0.461	0.462	0.469
2. Refi rate	3.84%	3.83%	3.82%	3.77%	3.78%	4.42%
3. Default rate	0.95%	0.99%	0.96%	0.55%	0.49%	0.08%
4. Household leverage	64.41%	64.50%	64.47%	65.80%	65.77%	58.83%
5. Mortgage debt to income	259.59%	264.80%	262.73%	280.24%	281.87%	232.74%
6. Loss-given-default rate	38.61%	36.70%	35.45%	35.12%	35.04%	35.56%
7. Loss Rate	0.40%	0.40%	0.38%	0.21%	0.19%	0.90%
<b>Intermediary</b>						
8. Bank equity ratio	7.09%	7.10%	7.35%	7.10%	7.17%	7.52%
9. Bank default rate	0.33%	0.29%	0.80%	0.19%	0.30%	0.38%
10. DWL of bank defaults	0.07%	0.06%	0.16%	0.04%	0.07%	0.07%
11. Deposits	2.454	2.503	2.477	2.650	2.661	2.203
<b>Prices</b>						
12. House Price	8.842	9.027	8.934	9.239	9.267	8.442
13. Risk-free rate	0.71%	0.73%	0.69%	0.75%	0.70%	0.96%
14. Mortgage Rate	1.46%	1.48%	1.54%	1.31%	1.29%	2.50%
15. Credit spread	0.75%	0.75%	0.85%	0.56%	0.59%	1.54%
16. Mortgage Expec. Excess Ret	0.34%	0.34%	0.46%	0.35%	0.39%	0.46%
<b>Welfare</b>						
17. Aggregate welfare	0.821	0.820	+0.13%	+0.08%	+0.03%	+0.53%
18. Value function, B	0.379	0.378	-0.55%	+0.47%	+0.16%	+1.61%
19. Value function, D	0.374	0.374	-0.04%	+0.07%	-0.08%	+0.29%
20. Value function, I	0.068	0.068	+4.79%	-2.13%	-0.11%	-4.12%
<b>Consumption and Risk-sharing</b>						
21. Consumption, B	0.359	0.358	-0.5%	+0.4%	+0.4%	+1.6%
22. Consumption, D	0.372	0.372	-0.6%	+0.1%	-0.3%	+1.4%
23. Consumption, I	0.068	0.069	+5.8%	-2.9%	-1.4%	-2.9%
24. Consumption gr vol, B	0.42%	0.39%	+408.8%	+14.8%	+43.2%	+56.4%
25. Consumption gr vol, D	1.11%	1.07%	-7.0%	-26.5%	-15.0%	-43.6%
26. Consumption gr vol, I	4.47%	4.46%	+402.9%	-53.5%	+149.3%	+81.4%
27. Wealth gr vol, I	0.035	0.033	+1483.0%	-10.0%	+369.0%	+425.3%
28. log (MU B / MU D) vol	0.025	0.024	+2.8%	-11.1%	-30.1%	-10.3%
29. log (MU B / MU I) vol	0.061	0.062	+143.6%	-35.2%	+52.8%	+8.3%

The table reports averages from a long simulation (10,000 periods) of the benchmark model with only strategic defaults (first column), then a set of models with liquidity default, whose labels follow the scheme in Tables 2 and 3. Rows 17-29 calculate percentage differences relative to the benchmark model with liquidity defaults in columns 3-6. All flow variables are quarterly.

Table 6: Results: Maintenance Rebates

	Baseline	Model with Maintenance Rebate				
	No Index	No Index	Aggregate	Local	Regional	Reg-Asym
<b>Borrower</b>						
1. Housing Capital	0.456	0.456	0.457	0.461	0.462	0.468
2. Refi rate	3.84%	3.84%	3.83%	3.81%	3.76%	4.43%
3. Default rate	0.95%	0.96%	0.91%	0.59%	0.48%	0.15%
4. Household leverage	64.41%	64.43%	64.26%	66.64%	66.09%	58.78%
5. Mortgage debt to income	259.59%	278.85%	272.57%	305.39%	300.69%	243.57%
6. Loss-given-default rate	38.61%	38.43%	37.41%	37.98%	37.33%	32.69%
7. Loss Rate	0.40%	0.40%	0.38%	0.24%	0.21%	0.92%
<b>Intermediary</b>						
8. Bank equity ratio	7.09%	7.10%	7.30%	6.88%	7.05%	7.64%
9. Bank default rate	0.33%	0.23%	0.83%	0.14%	0.54%	0.31%
10. DWL of bank defaults	0.07%	0.05%	0.17%	0.03%	0.13%	0.06%
11. Deposits	2.454	2.636	2.570	2.892	2.840	2.304
<b>Prices</b>						
12. House Price	8.842	9.498	9.278	9.953	9.829	8.863
13. Risk-free rate	0.71%	0.72%	0.63%	0.64%	0.55%	0.99%
14. Mortgage Rate	1.46%	1.45%	1.50%	1.24%	1.26%	2.57%
15. Credit spread	0.75%	0.74%	0.87%	0.60%	0.72%	1.58%
16. Mortgage Expec. Excess Ret	0.34%	0.33%	0.49%	0.36%	0.51%	0.47%
<b>Welfare</b>						
17. Aggregate welfare	0.821	0.872	+0.12%	+0.43%	+0.59%	+0.24%
18. Value function, B	0.379	0.399	-0.70%	+1.11%	+0.72%	+0.95%
19. Value function, D	0.374	0.401	-0.08%	+0.04%	-0.05%	+0.11%
20. Value function, I	0.068	0.071	+5.84%	-1.16%	+3.40%	-3.01%
<b>Consumption and Risk-sharing</b>						
21. Consumption, B	0.359	0.383	-0.4%	+0.8%	+0.6%	+0.7%
22. Consumption, D	0.372	0.398	-1.1%	-0.4%	-1.4%	+1.2%
23. Consumption, I	0.068	0.073	+6.8%	-0.7%	+5.2%	-1.3%
24. Consumption gr vol, B	0.42%	0.53%	+290.9%	+38.6%	+224.5%	+65.0%
25. Consumption gr vol, D	1.11%	1.08%	-10.1%	-13.2%	-4.3%	-45.1%
26. Consumption gr vol, I	4.47%	4.73%	+401.6%	-40.0%	+354.5%	+104.6%
27. Wealth gr vol, I	0.035	0.034	+1498.6%	+6.9%	+937.0%	+568.9%
28. log (MU B / MU D) vol	0.025	0.026	-10.1%	+4.0%	-22.2%	-13.8%
29. log (MU B / MU I) vol	0.061	0.062	+154.9%	-19.1%	+137.6%	+29.3%

The table reports averages from a long simulation (10,000 periods) of the benchmark model without maintenance rebates (first column), then a set of models where maintenance is rebated, whose labels follow the scheme in Tables 2 and 3. Rows 17-29 calculate percentage differences relative to the benchmark model with liquidity defaults in columns 3-6. All flow variables are quarterly.