# Winners and Losers When Interest Rates Change*

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**Abstract**

Real rates declined by more than 4% points between 1980 and 2023 driving large capital gains on long-lived assets. Households that rely on their financial wealth to finance future consumption need more wealth to fund the same consumption plan after rates have declined. To be hedged against interest rate risk, households need to match the duration of their portfolio to the duration of a claim on their future consumption in excess of labor income. We find that young and poor US households were worse off when rates declined, because they had too little duration in their portfolios. Older and wealthy US households were better off. We characterize the compensated financial wealth distribution implied by full hedging and compare it to the actual one.

**JEL:** E21, E25, E44, G12

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Over the post-war period, interest rates and financial wealth inequality have displayed a remarkable negative correlation. Real interest rates fell by 4% points from 1983 until 2021 while the top-10% share of financial wealth rose from 63% in 1983 to 71% in 2021. Over the same period, the top-1% share of wealth grew from 24% to 35%. Greenwald, Leombroni, Lustig, and Van Nieuwerburgh (2025), henceforth GLLV, show that much of the rise in wealth inequality can be accounted for by the combination of persistently declining interest rates and cross-sectional heterogeneity in the interest rate sensitivity of household portfolios. They use microdata to show that wealthy households have portfolios with much higher duration than poor households, because the former hold a larger share of their portfolio in high-duration assets such as stocks and private business wealth while the latter hold a larger share of their portfolio in short-duration assets such as deposits and vehicles. GLLV (2025) asks the positive question of what share of the observed rise in financial wealth inequality can be accounted for by falling interest rates. To that end, they compute the *repriced* financial wealth distribution, which revalues households' assets in light of the observed changes in interest rates given observed portfolio durations.

This paper asks the normative question: What are the implications of rising financial wealth inequality for inequality in the consumption possibilities that determine welfare. It is by no means obvious whether the changes in measured financial wealth inequality reflect changes in consumption possibilities (welfare), or simply represent revaluations of the same consumption plans, i.e., "paper gains and losses" (Moll, 2020). We study when rising financial wealth inequality is associated with rising welfare inequality, conceptually. We also study how large the differences are between the two concepts in a model that fits basics facts on the evolution income and wealth inequality for the U.S. economy since the 1980s.

To do so, we compute the *compensated* financial wealth distribution. It tracks the level of financial wealth that each household would require to keep its consumption plan unchanged following a movement in rates. Intuitively, when interest rates come down, buying the old consumption stream becomes more expensive. The price of the annuity whose payouts mimic the consumption plan goes up when rates go down. At the same time, the household's labor income stream, its human capital, becomes more valuable. If the decline in rates increases the cost of the consumption plan by more than the increase in human wealth, the household needs an increase of financial wealth to be as well off as she was prior to the rate decline. The increase in financial wealth she experiences depends on the duration of her financial portfolio. The household is fully hedged when the increase in financial wealth exactly equals the increased cost of her net consumption plan. Put more succinctly, household consumption is unaffected by a change in rates if and only if the compensated and repriced distributions are identical. In an economy where households are fully-hedged against rate changes, a reduction in interest rates may well result in a large rise in financial wealth inequality, but it is not associated with a rise in welfare inequality.

We show that rising financial wealth inequality paired with constant welfare inequality is the natural outcome in a large class of incomplete markets models where households are infinitely-lived, ex-ante identical, but ex-post heterogeneous because they receive different labor income shocks. In those models, households with low financial wealth are those who have encountered a sequence of bad labor income shocks. If labor income is persistent but mean-reverting, their labor income is low today and in the near future but higher in the distant future. This pattern makes the duration of their labor income stream high. Since households are risk averse, they optimally smooth consumption inter-temporally by borrowing to turn some of that higher future labor income into higher consumption today. As a result, low-wealth agents tend to have a duration of the consumption stream that is lower of that of their labor income stream. They have low excess consumption duration. The same economic forces of mean-reversion in income and consumption smoothing imply that rich agents have high excess consumption duration. In the population, there is a positive covariance between the level of financial wealth and the duration of excess consumption. We show that when this covariance is positive, lower interest rates increase financial wealth inequality when households are fully hedged.

While this standard Bewley model serves as an interesting benchmark, we consider a richer model which adds life-cycle dynamics. Households are no longer ex-ante identical because they differ in age. The result that a positive cross-sectional covariance between the level of financial wealth and the duration of excess consumption is a necessary and sufficient condition for lower interest rates to increase financial wealth inequality when households are fully hedged continues to hold in this richer model. However, the life-cycle model adds an important new force: saving for retirement. Life-cycle savings motives imply that most households have negative excess consumption in middle age, as they save for retirement, followed by positive excess consumption when they are in retirement. The excess consumption duration is highest for young households, and declining in age. Since younger households tend to be poorer, this retirement savings force leads to a negative cross-sectional covariance between financial wealth and the duration of excess consumption.

Which of the two opposing forces highlighted above, the history of stochastic income realizations and the life-cycle effect of saving for retirement, dominates is a quantitative question. We build a quantitative model that is calibrated to match the top-10% financial wealth share in 1983. It features rich idiosyncratic income risk calibrated to the Panel Survey of Income Dynamics, a superstar income state, and a bequest motive. We find that the retirement savings force dominates the consumption smoothing of income shocks force. The covariance between financial wealth and the duration of excess consumption in the population in the model's equilibrium is negative. The main proposition then implies that, in a perfectly-hedged lifecycle economy where rates do not affect welfare inequality, we would see financial wealth inequality fall following a decline in interest rates. To ensure that all households could afford their pre-shock consumption plans, we would have needed to see the top-10% financial wealth share decline bypp over our sample period.

This is the opposite pattern as the sharp rise in financial wealth inequality observed in the data since the 1980s, which is quantitatively accounted for by the model's repriced wealth distribution which uses the observed portfolio durations. In other words, the data are at odds with perfect hedging. Households' actual financial wealth portfolios do not insulate them against the effect of rate changes on their consumption plan. The compensated and repriced financial wealth distribution are far away from each other, implying major effects of rate changes on welfare inequality. This large mismatch between the compensated and repriced distribution implies that falling interest rates have not merely adjusted financial wealth "on paper." Instead, there are winners and losers from falling rates, with life-cycle dynamics playing a key role. The young need to save in middle age and dissave in retirement, giving them a high duration of excess consumption. However, young households on average have little financial wealth accumulated and not enough financial wealth duration to fully hedge this excess consumption plan. As a result, young households see consumption possibilities contract when rates fall. Intuitively, they find it more challenging to accumulate wealth for retirement in the absence of high returns. In contrast, older households who have already accumulated wealth benefit when rates fall, earning large capital gains on their assets that more than offset lost investment income going forward. We also find that lower-wealth households, in addition to the young, disproportionately lose from lower rates.

This heterogeneity in the effect of rate changes across the age distribution implies that the total impact of falling rates on an individual household's consumption and welfare depends not only on how much rates fall during its lifetime, but also on exactly when in its life cycle these falls occur. We use the model to study which cohorts gained and lost from the fall in rates between the 1980s and 2010s. We find that households born in the 1920s through the 1940s gained substantially from falling rates, as these households had largely accumulated their peak financial wealth by the time rates began to fall in the 1980s. In contrast, households born in the 1960s or later generally lost from declining rates. This drop in consumption is severe for recent cohorts, with households born in the 2000s losing more than 8% of lifetime consumption at birth due to the decline in rates.

**Related Literature.** Our paper contributes to the literature on the rise in wealth inequality (Piketty and Saez, 2003; Piketty, 2015; Alvaredo, Chancel, Piketty, Saez, and Zucman, 2018). GLLV (2025) shows that declines in real rates of interest, or more broadly declines in expected returns across financial asset classes, combined with heterogeneity in households' portfolio duration goes a long way towards accounting for the observed rise in financial wealth inequality. This explanation complements other explanations that have emphasized return heterogeneity within asset classes, differential savings rates, and a skewed and persistent distribution of stochastic earnings (e.g., De Nardi, 2004; Piketty and Zucman, 2015; Benhabib, Bisin, and Luo, 2017; Cox, 2020; Fagereng, Guiso, Malacrino, and Pistaferri, 2020; Bach, Calvet, and Sodini, 2020; Hubmer, Krusell, and Smith, 2020). Our model features a skewed and persistent earnings distribution and allows in time-varying idiosyncratic income risk. The duration heterogeneity mechanism generates the fact that the return on the financial wealth is increasing in the level of financial wealth in samples with declining interest rates. This paper takes the duration explanation for the rise in wealth inequality as given and studies its normative implications.

Auclert (2019) shows that how transitory monetary policy shocks impact aggregate consumption and household welfare depends on how it redistributes wealth across households with different propensities to consume. For this transitory shock, the sufficient statistic is unhedged interest rate exposure (URE) — the net difference between assets and liabilities that pay in the future rather than today — which does not distinguish between future cash flows arriving at different times (e.g., two-year vs. five-year zero coupon bonds). In contrast, we consider the impact of a *permanent* shock to interest rates on consumption and wealth *inequality*, for which the exact timing of the cash flows is critical. In our context, the sufficient statistics are the duration of financial wealth and excess consumption, which for a permanent shock can drive variation among households with the same URE.[1] We see these works, studying distinct objects following interest rate shocks of different persistence, as highly complementary.

In contemporaneous work, Fagereng, Gomez, Gouin-Bonenfant, Holm, Moll, and Natvik (2025) also study changes in wealth and welfare, focusing on the case of Norway. Housing wealth is a much larger share of total wealth there than in the United States. Conceptually, they focus on changes in net positions while we emphasize the duration of excess consumption plans.

Catherine, Miller, Paron, and Sarin (2023) build on our work to study how well hedged different households are to interest rate changes, emphasizing the role of Social Security benefits, which Catherine, Miller, and Sarin (2025) shows affects the measurement of financial wealth inequality. Human wealth in our model incorporates all forms of income, including Social Security benefits. Our results on consumption and the compensated distribution allow us to speak to what measured inequality *should be* to keep consumption opportunities constant.

For an alternative to our duration measure, recent work by Fagereng et al. (2025) proposes using future net asset purchases to gauge the welfare effects of real rate declines. They apply this measure to Norwegian household-level data, where housing is the dominant form of wealth, and show that their flow-based approach is equivalent to our duration approach following a permanent shock to interest rates. While this flow-based approach is ideal to study the effects of total asset price changes, our work considers the contribution of a *single factor* driving asset prices — falling interest rates — on inequality over time. This allows us to use duration as a sufficient statistic, making our measurement exercise possible in a US context that lacks the detailed transaction data used by Fagereng et al. (2025). Last, our model allows us to study the dynamic effects of

[1]The working paper version of Auclert (2019) features numerical exercises showing that the impact of persistent monetary policy shocks on aggregate consumption depends on asset durations. We complement this work by formalizing and estimating sufficient statistics for the response of consumption and wealth inequality to permanent shocks.

changes in valuations over time, as they propagate through consumption and savings behavior, rather than on impact. We again view these differing research questions and methodologies as highly complementary.

In additional work, Gomez and Gouin-Bonenfant (2020) also study the effects of lower interest rates on inequality, through their impact on the cost of raising new capital for entrepreneurs. This investment-based channel operates through growth in real assets and cash flows, complementing our duration-based mechanism operating through financial revaluation of cash flows. Kuhn, Schularick, and Steins (2020) use novel microdata to document that heterogeneity in the shares of equity and housing in household portfolios drove much of the rise in inequality since the 1970s. We build on this work by constructing new and detailed statistics on the duration of financial wealth for US households, which summarize how declining interest rates over this period translated into changes in the prices of these assets. Our approach is also closely linked to that of Doepke and Schneider (2006), who focus on the distributional consequences of inflation. We follow these authors in using household-level portfolio data to measure the exposure to the shock of interest, but focus on the effects of changes in long-term real rates rather than inflation.

Additional recent work analyzes the measurement challenges for private business income and wealth.[2] In our empirical work, we advance this agenda by producing several new measures of private business wealth that combine microdata from the Survey of Consumer Finances with asset pricing data from a range of data sources. We perform various exercises to establish the robustness of the results to the specifics of the private business wealth duration estimation.

Last, our paper links to the rich literature on the mechanisms behind the decline in interest rates over our sample.[3] Our conclusions regarding the *consequences* of low interest rates for wealth inequality stemming from asset revaluations should not depend on this fundamental source, as a change in discount rates will affect the valuation of a fixed stream of cash flows in the same way regardless of its ultimate cause. We thus view our work as complementary to, but distinct from, this important literature.

**Overview.** The rest of the paper is organized as follows. Section 1 derives the link between cash flow duration and financial wealth inequality, and derivs the main proposition. Section 2 calibrates the model and analyzes its stationary distribution. Section 3 discusses the main quantitative results for the repriced and compensated wealth distributions, respectively. Section 4 computes the impact of rates on total wealth by birth cohort. Section 5 concludes.

The paper also includes a comprehensive appendix. Appendix B contains proofs of our propositions in the main text.

[2] See e.g., Kopczuk (2017); Saez and Zucman (2016); Piketty, Saez, and Zucman (2018); Smith, Yagan, Zidar, and Zwick (2022.); Kopczuk and Zwick (2020).

[3] See e.g., Bernanke (2005); Caballero, Farhi, and Gourinchas (2008); Summers (2014); Eggertsson and Mehrotra (2014); Eichengreen (2015); Gutiérrez and Philippon (2017); Mian, Straub, and Sufi (2020).

# 1 Model

To develop theoretical and quantitative insights on how changes in interest rates affect the distribution of wealth, we develop a simple life-cycle model with idiosyncratic labor income risk that connects interest rates, duration, and wealth inequality in a transparent fashion. In order to straightforwardly apply the exact path of rates that occurred in the data, we use a partial equilibrium model (alternatively, a small open economy) where interest rates are taken as given, and abstract from the structural mechanisms or shocks that caused the interest rate to fall. Instead, we analyze the relationship between wealth, interest rates, and consumption as an accounting identity. Regardless of the underlying cause, the duration measures we use accurately describe the change in financial wealth and consumption possibilities due to the observed decline in interest rates relative to a counterfactual world where interest rates had not fallen but all other variables had evolved identically. Our work thus provides a robust quantitative measure that complements work on the underlying forces driving changes in interest rates over this period. In the next section, we describe a model that endogenizes interest rates.

## 1.1 Duration

We begin by defining the duration of an asset and a portfolio. We consider the response to an unexpected and permanent change in the interest rate from $R$ to $\tilde{R} = R\exp(\varepsilon)$ for some shock $\varepsilon$, where $\varepsilon < 0$ corresponds to a fall in rates. Throughout the paper we will use tildes (e.g., $\tilde{R}$) to indicate updated values following an interest rate shock, while equivalent variables without tildes indicate pre-shock values. This is a special case of GLLV (2025), who consider not only a permanent shock model for interest rates, but also a transitory shock model and a combination of permanent and transitory shocks. Since they show that the quantitative results are similar across interest rate models, we focus here on the permanent shock model.

**Definition 1.** Assume that the annualized discount rate is equal to $r$ at all maturities, and let $R = 1 + r$. Given a sequence of cash flows $\{x_t\}$, define its present value at $t = 0$ by

$$P_0 = \sum_{t=0}^{\infty} R^{-t} x_t \tag{1}$$

The *cash flow duration* (or simply, duration) of this stream of cash flows is defined by

$$D \equiv \frac{\sum_{t=0}^{\infty} t \times R^{-t} x_t}{P_0}. \tag{2}$$

In words, cash flow duration is the weighted average of the time remaining to receive each cash flow, weighted by the share of the asset's value attributable to that cash flow. The duration of a zero-coupon bond equals the maturity of the bond.

Duration is the key concept that links interest rate changes to asset and portfolio revaluation. Consider a sequence of cash flows $\{x_t\}$ with present value $P_0$. Then

$$\frac{\partial \log P_0}{\partial \log R} = -D$$

where $D$ satisfies (2). For a small shock $\varepsilon \to 0$, this implies the approximate revaluation

$$\tilde{P}_0 \simeq P_0 \exp(-D \times \varepsilon) \simeq P_0(1 - D \times \varepsilon). \tag{3}$$

For a portfolio of assets indexed by $k$, equation (3) holds using the value-weighted duration

$$D^{VW} \equiv \sum_k \omega(k) D(k) \tag{4}$$

where $\omega(k)$ is the share of the portfolio's value in asset $k$, and $D(k)$ is the duration of asset $k$.

### 1.2 Model Structure

**Demographics.** The economy is populated by a continuum of households. Households transition through a life cycle, where age $j$ varies from 0 to $J$. Households survive from age $j$ to age $j+1$ with probability $\phi_j$, with $\phi_J = 0$. When a household dies, it is replaced by a newborn household ($j = 0$), which inherits its remaining assets as a bequest.

**Endowments.** Each household $i$ of age $j$ receives exogenous labor income given by $y_j(z)$, where $z$ is a household-specific (idiosyncratic) stochastic process.

**Asset Technology.** Households trade a complete set of bonds offering fixed cash flows at future dates.[4] Without loss of generality, we restrict attention to zero coupon bonds, where a zero coupon bond with maturity $m$ promises one unit of the numeraire in $m$ periods. We denote holdings of each bond as $x_m$, and its price as $q_m$. Markets are incomplete in that households cannot contract on their idiosyncratic income realizations.

We consider an economy in steady state, so that prices $\{q_m\}$ are expected to hold in all future periods. We further assume that the one-period bond is traded on a global market in which our model economy is a price taker, so that its interest rate takes the exogenous value $R$. If we normalize $q_0 = 1$, the absence of arbitrage opportunities requires $q_m = R^{-m}, \forall m$ in steady state.

[4]The model with aggregate shocks in Appendix C allows for asset payoffs that depend on the aggregate state.

**Household Problem.** Given start-of-period bond holdings $x$, labor income $y$, and bond prices $q$, a household of age $j$ chooses consumption $c$ and bond holdings $x'$ to solve the recursive problem

$$V_j(x;z) = \max_{c,x'} \underbrace{\frac{c^{1-\gamma}}{1-\gamma}}_{\text{flow utility}} + \underbrace{\phi_j \beta \mathbb{E}\left[V_{j+1}(x';z') \,\middle|\, z\right]}_{\text{continuation value}} + \underbrace{(1-\phi_j)\chi \frac{\left(\sum_{m=1}^{M} q_{m-1} x'_m\right)^{1-\gamma}}{1-\gamma}}_{\text{bequest utility}} \tag{5}$$

subject to the budget constraint,

$$c \leq y_j(z) - \underbrace{\sum_{m=1}^{M} \left(q_m x'_m - q_{m-1} x_m\right)}_{\text{net saving}}$$

and the borrowing constraint $\sum_m q_m x'_m \geq 0$, which rules out negative bequests. While past work has shown empirical benefits from using non-homothetic bequest functions (De Nardi, 2004; Straub, 2019), our use of a homothetic bequest function offers large gains in tractibility for our theoretical and quantitative analysis. We note that because these non-homothetic bequest functions imply greater savings by the rich following wealth gains, and hence greater amplification and persistence of wealth inequality, our results can be seen as, if anything, conservative.

**Household Optimality.** Each of the $m$ optimality conditions for bond holdings $x'_m$ collapses to the same Euler equation modified to include bequest utility:

$$c^{-\gamma} = R\left\{\phi_j \beta \mathbb{E}\left[(c')^{-\gamma} \,\middle|\, z\right] + (1-\phi_j)\left(\sum_{m=1}^{M} q_{m-1} x'_m\right)^{-\gamma}\right\}. \tag{6}$$

These optimality conditions do not uniquely identify the portfolio holdings, since households expect to receive the same holding period return $R$ on all bond maturities. Instead, only the household's total financial wealth $\theta \equiv \sum_{m=1}^{M} q_{m-1} x_m$ and total savings $s \equiv \sum_{m=1}^{M} q_m x'_m$ matter for the household's problem in a steady state where interest rates do not change. Given this indifference, we will later assign each household a unique portfolio $\{\hat{x}_m\}$ that matches its empirically predicted duration given its position in the wealth distribution.

**Financial Wealth.** Using the results above, we can simplify the household's problem using total financial wealth $\theta$ as a single state variable. Since next period financial wealth $\theta'$ is given by

$$\theta' = \sum_{m=1}^{M} q_{m-1} x'_m = R^{-1} \sum_{m=1}^{M} q_m x'_m$$

we can rewrite (5) as the optimization problem

$$V_j(\theta; z) = \max_{c,\theta'} \frac{c^{1-\gamma}}{1-\gamma} + \phi_j \beta \mathbb{E}\left[V_{j+1}(\theta'; z') \;\middle|\; z\right] + (1-\phi_j)\chi \frac{(\theta')^{1-\gamma}}{1-\gamma}$$

subject to the budget constraint

$$c \leq y_j(z) + \theta - R^{-1}\theta'. \tag{7}$$

and the no borrowing condition $\theta' \geq 0$, yielding the single optimality condition

$$c^{-\gamma} = R\left\{\phi_j \beta \mathbb{E}\left[(c')^{-\gamma} \;\middle|\; z\right] + (1-\phi_j)(\theta')^{-\gamma}\right\}. \tag{8}$$

Although households believe the economy will remain in steady state forever, we apply unanticipated shocks to interest rates that will revalue financial assets. In this case, although households believed (7) would hold, the actual next period financial wealth is updated according to

$$\tilde{\theta}' = \sum_{m=1}^{M} \tilde{q}_{m-1} x'_m$$

where $\{\tilde{q}_m\}$ is the updated set of bond prices conditional on the new realized interest rate.

### 1.3 Main Result: Consumption Effects of Interest Rate Changes

While the previous section clarified the impact of interest rates on financial wealth, it is by no means obvious whether these changes in measured financial wealth reflect changes in consumption possibilities (welfare), or whether they merely represent revaluations of the same consumption plans (paper gains and losses). To distinguish the two, we iterate forward (7) to obtain

$$\theta_0 = \mathbb{E}_0\left\{\sum_{t=0}^{T-1} R^{-t}(c_t - y_t) + R^{-T}\theta_T\right\} \tag{9}$$

where $T$ is the (stochastic) death date, measured in number of periods from the present ($t = 0$), and $\theta_T$ is the final bequest of this household, which depends on the histories of $c_t$ and $y_t$, as well as the realization of $T$. Financial wealth is thus equal to the present value of future excess consumption, defined as consumption minus income $c_t - y_t$, plus the present value of the bequest $\theta_T$.

For this section, we consider the infinite-horizon limit with vanishing mortality risk, so that

$$\theta_0 = \mathbb{E}_0\left\{\sum_{t=0}^{\infty} R^{-t}(c_t - y_t)\right\}. \tag{10}$$

With this identity in hand, we turn to our main theoretical insight regarding the link between interest rates and consumption possibilities.

**Proposition 1.** Assume that the interest rate $R$ and discount factor $\beta$ both unexpectedly and permanently change to $\tilde{R} = R\exp(\varepsilon)$ and $\tilde{\beta} = \beta\exp(-\varepsilon)$. Define the duration of financial wealth by $D^{\theta}$, and the duration of future excess consumption $D^{c-y}$ by

$$D^{\theta} \equiv \frac{\sum_{m=1}^{M} m \times q_m x_m}{\sum_{m=1}^{M} q_m x_m}, \qquad D^{c-y} \equiv \frac{\mathbb{E}_0\left\{\sum_{t=0}^{\infty} t \times R^{-t}(c_t - y_t)\right\}}{\mathbb{E}_0\left\{\sum_{t=0}^{\infty} R^{-t}(c_t - y_t)\right\}}.$$

Let $\tilde{c}_t$ denote the household's consumption plan following the change in rates. Then for a decline in rates ($\varepsilon < 0$):

(a) The set of consumption allocations the household can afford expands if $D^{\theta} > D^{c-y}$, contracts if $D^{\theta} < D^{c-y}$, and is unchanged if $D^{\theta} = D^{c-y}$.

(b) If, following the shock, a household has exactly enough financial wealth $\tilde{\theta}_0$ to afford its pre-shock consumption plan $\{c_t\}$, then this consumption plan remains optimal.

(c) Household consumption is unchanged following a shock to interest rates ($c_t = \tilde{c}_t$, $\forall t$) if and only if $D^{\theta} = D^{c-y}$.

The proof can be found in Appendix B.1. Part (a) shows that whether changes in interest rates expand or contract a household's consumption possibilities depends not on the absolute level of its financial wealth duration, but on how it compares to the duration of that household's lifetime excess consumption. While financial wealth is always equal to present value of future excess consumption by the budget identity (10), the two may be differentially exposed to the same interest rate shock, much like a bank with a maturity mismatch of assets and liabilities. As a result, even if a household gains financial wealth from a decline in rates, it can still see its consumption possibilities contract if the cost (present value) of its pre-shock excess consumption plan rises by more than its financial wealth.

This result builds on Auclert (2019), but is different. Auclert (2019) shows that for perfectly transitory change in the real rate, the consumption response depends on unhedged interest rate exposure (URE). Mapped into the language of this section, this result states that the response depends on whether the present value of future excess consumption or future net cash flows from the household's financial portfolio is greater. This is a sufficient statistic because a perfectly transitory change in the short-term discount rate changes the cumulative discounting of all future cash flows by a constant proportional amount. In our setting with permanent shocks, the exact timing of the cash flows matters, as cash flows further in the future are more affected by a permanent change in rates than cash flows closer to the present. As a result, URE is no longer a sufficient statistic. For example, a household with a portfolio of five-year bonds has the same URE as an otherwise identical household holding two-year bonds, but will experience much larger gains following a decline in rates. Our sufficient statistic of duration accounts for these timing effects, allowing us to extend these results to a setting with permanent shocks.

Part (b) of Proposition 1 shows that, as long as a household can exactly afford its pre-shock consumption plan, it will still find this plan optimal, and choose it in equilibrium. This follows directly from (8) in the absence of bequests, and the fact that $\beta R = \tilde{\beta}\tilde{R}$. We will exploit this result in Section 3 to construct a counterfactual wealth distribution that leaves consumption approximately unchanged following a shock .

Part (c) follows immediately, since the pre-shock consumption plan is optimal if and only if it is exactly affordable after the shock, which occurs if and only if $D^{\theta} = D^{c-y}$. Households whose financial wealth and excess consumption durations are perfectly aligned ($D^{\theta} = D^{c-y}$) do not change their consumption plans following a decline in rates. In such a "perfectly hedged" economy, any changes in financial wealth inequality that arise as a result of the interest rate changes would therefore reflect only "paper" gains, while keeping consumption/welfare inequality unchanged.

Proposition 1 assumes zero mortality risk. Although part (a) would hold absent this assumption, parts (b) and (c) would not. This is because we follow the convention of measuring a household's bequest in current value (i.e., dollars). Thus, even if a household's consumption plan is fixed, falling rates that increase the present value of future cash flows will mechanically increase the value of financial wealth the household maintains along its consumption path, thus increasing the "size" of bequests. This issue highlights a potential downside of typical bequest parameterizations, since households receive more utility from a more "valuable" bequest when rates fall, even though the heirs receiving it will not be able to afford larger consumption allocations. Regardless, the quantitative discrepancy should be small, particularly for young households whose bequests are far in the future on average. The quantitative model will consider stochastic mortality and bequests.

In this class of Bewley models withe ex-ante identical households, households with low financial wealth have encountered a bad history of labor income shocks. If labor income is persistent but mean-reverting, their labor income is low today and in the near future relative to labor income in the far future. This pattern makes the duration of their labor income stream high. But since the household is smoothing consumption inter-temporally, it wishes to bring consumption forward in time and the duration of the optimal consumption stream is shorter: $D^c < D^y$. As a result, low-wealth agents tend to have low duration of their excess consumption plan. Conversely, rich agents have high labor income and high excess consumption duration: $D^c > D^y$. Their excess consumption stream has high duration. Inter-temporal consumption smoothing is the force that drives the positive covariance between the level of financial wealth and the duration of excess consumption in a Bewley model where heterogeneity results from income shock realizations.

Proposition 4 in GLLV (2025) shows that for a small negative change in rates ($\varepsilon < 0$):

(a) The growth in household wealth due to revaluation, $\tilde{W}^i / W^i$, has a positive cross-sectional covariance with wealth $W^i$ if and only if the generalized duration of wealth, $\mathcal{D}^i$, has a positive covariance with the level of wealth $W^i$. A necessary and sufficient condition for this positive covariance is that aggregate (value-weighted) wealth duration exceeds average (equal-weighted) generalized duration in the pre-shock economy.

(b) The top-$\alpha$ share of wealth $S^\alpha$ increases if and only if $\mathcal{D}^{top}$, the value-weighted generalized duration for the top-$\alpha$ wealthiest share of households exceeds $\mathcal{D}^{bottom}$, the value-weighted generalized duration for the bottom $1-\alpha$ share of the wealth distribution (or equivalently, exceeds the overall value-weighted generalized duration $\mathcal{D}$). The change in the top-$\alpha$ wealth share is approximated by

$$d\tilde{S}^\alpha \simeq -S^\alpha(1 - S^\alpha)(\mathcal{D}^{top} - \mathcal{D}^{bottom})\varepsilon. \tag{11}$$

It follows from Proposition 4 in GLLV (2025) that the decline in rates increases the cost of the aggregate excess consumption plan (value-weighted average across households) by more than the cost for the equally-weighted average household, and that it increases financial wealth inequality. Put simply, in a Bewley model where all households are exactly equally well off after the change in interest rates, i.e., they are perfectly hedged, financial wealth inequality increases when rates go down. Low-financial wealth households in a Bewley model have high-duration human wealth, which provides a natural interest rate hedge. High financial-wealth households have low-duration human wealth and need a larger increase in financial wealth when rates decline to be able to afford the old consumption plan.

Seen through the lens of this model, observing a rise in financial wealth inequality following a decline in interest rates is exactly what one would expect to see. Crucially, it would not indicate any change in welfare inequality since all households are perfectly hedged. This result holds regardless of the source of the decline in interest rates (recall rates are endogenous in this model), and even in the presence of borrowing constraints and aggregate risk.

### 1.4 The Role of Life-Cycle Considerations

The insights of Proposition 4 in GLLV (2025) apply more broadly to a richer model with ex-ante heterogeneity across households, for example because agents go through a life cycle and differ by age. The sufficient condition remains the cross-sectional covariance of financial wealth and the duration of excess consumption. If that covariance is positive, lower rates increase financial wealth inequality while leaving welfare inequality unaffected. A positive covariance implies that the duration of excess consumption $D^{c-y}$ is higher for wealthy households than poor households.

In the previous section, we discussed an important force that pushes towards a positive covariance: consumption smoothing in the presence of persistent, mean-reverting labor income risk. In the presence of a life cycle, however, there is a second force which pushes in the opposite direction, towards a negative cross-sectional covariance. In life cycle economies, the young are typically the least wealthy. At the same time, life-cycle savings motives imply that most households have negative excess consumption in middle age, as they save for retirement, followed by positive excess consumption in retirement. Thus, $D^{c-y}$ is highest for young households, and decreasing in age. This force pushes down the cross-sectional covariance of financial wealth and the duration of excess consumption.

The two opposing forces highlighted above—the history of stochastic income realizations and the life-cycle effect of saving for retirement— appear in virtually any model of household inequality. The question of which one dominates, and the ultimate implications for wealth inequality under perfect hedging, is a quantitative one. In the following sections, we answer it using a standard calibration of a workhorse life-cycle incomplete markets model.

As we shall see, the life-cycle effect dominates the income risk effect, and the covariance is negative. Proposition 4 in GLLV (2025) then implies that we should see financial wealth inequality *fall* following a decline in interest rates in a perfectly hedged economy. This prediction is at odds with the large rise in financial wealth inequality since the 1980s in the data. The natural conclusion is that, in the real world unlike in the model, households are *not* fully hedged. The change in interest rates then has a different effect on households' financial wealth, the extra wealth they have, than on their excess consumption plan, the extra wealth they need. Such a discrepancy implies that there are real consumption effects from interest rate changes, and that these effects are unequally distributed across households. The goal of the next section is to quantify these heterogeneous consumption effects.

## 2 Model Quantification

To measure the effect of changing interest rates on wealth inequality, our model must produce quantitatively realistic responses. In this section, we parameterize our model, calibrate it to match data on both household financial wealth durations, income risk, and wealth shares, and present the properties of the model's stationary economy.

### 2.1 Calibration

**Preferences and Mortality Risk.** We calibrate the model's mortality risk via the survival probabilities $\phi_j$ to match Social Security Actuarial tables. Since we model households, we take the average of the male and female mortality rates, weighted by the proportion alive at each age.

We set $\gamma$, the risk aversion of the households and the curvature of the bequest function, to a standard value of 2. We set the time discount factor to $\beta = 0.949$ to target a ratio of net wealth to disposable labor income in 1983 of 6.79.[5] We set the bequest utility parameter $\chi = 7.894$ to match a ratio of bequests to GDP of 8.8%, following Auclert, Malmberg, Martenet, and Rognlie (2021).

**Interest Rates** We set the steady-state interest rate $R$ to match the 10-year real rate in 1983. The interest rate in the data comes from an economy with growth. As shown in Appendix C.3.2, the mapping between the interest rate in our stationary economy and that in the growing economy is:

$$R = \frac{R_g}{G}, \qquad \beta = G \times \beta_g \tag{12}$$

where $R_g$ and $\beta_g$ are the gross interest rate and time discount factor in the growing economy, $R$ and $\beta$ are the corresponding values in our stationary economy, and $G$ is the gross growth rate (adjusted for a Jensen effect and a risk premium). Given average log growth of 1.91%, the observed rate in 1983 of $R_g = 4.94\%$ implies $R = 3.04\%$.

**Financial Duration** As households in the model go through their life cycle and experience changes in financial wealth, their financial duration is updated according to

$$\widehat{D}_i^{\theta} = \hat{\alpha} + \hat{\beta} Age_i + \sum_j \hat{\gamma}_j NetWealthBin_{i,j}. \tag{13}$$

**Regular Income Parameters.** The labor income process consists of a regular component and a superstar component. The regular income process for household $i$ of age $a$ at time $t$ that is not currently in the superstar state takes the form, standard in the literature, given by:

$$\log\left(y^i_{t,a}\right) = m_t + \chi' X^i_t + z^i_t, \tag{14}$$

$$z^i_{t+1} = \alpha_i + \eta^i_{t+1} + \nu^i_{t+1}, \tag{15}$$

$$\eta^i_{t+1} = \rho \eta^i_t + u^i_{t+1}, \tag{16}$$

where $m_t$ is a year-fixed effect and $X^i_t$ is a vector of household characteristics that includes a cubic function of age.[6] We normalize the mean of the age profile to unity during working life.

The stochastic income component $z^i_t$ contains a household-fixed effect $\alpha^i$, a persistent component $\eta^i_{t+1}$, and an i.i.d. component $\nu^i_{t+1}$. We set $\mathbb{E}[\nu^i] = E[u^i] = \mathbb{E}[\alpha^i] = \mathbb{E}[\eta^i_0] = 0$, while $Var[\nu^i] = \sigma^2_\nu$, $Var[u^i] = \sigma^2_u$, $Var[\alpha^i] = \sigma^2_\alpha$, and $Var[\eta^i_0] = \sigma^2_{\eta,0}$. To allow for lower income risk

[5] Net wealth is measured as the net worth of households and nonprofits in the FAUS (Table B.101). Disposable labor income is household income net of personal taxes, rental, interest, and dividend income, and one half of proprietor's income in the National Income and Product Accounts.

[6] Our results are similar if we estimate the year fixed effect and the age profile separately for groups of households that depend on education (college completion or not), race (white or non-white), and gender of head of household (8 groups total). Since it makes little difference for our main results, we assume ex-ante identical households for simplicity.

during retirement, we re-estimate (A.1) – (A.3) separately for households above and below age 65, and assume that model households face income risk that switches when they turn 65. The parameters are estimated by GMM using PSID data from 1970 until 2017, as detailed in Appendix A. Figure A1 plots the deterministic life-cycle income profile.

The literature typically estimates (A.1) – (A.3) on labor income for males between ages 25 and 55. We deviate from this practice by: (i) considering a broader income concept, (ii) modeling the entire life-cycle from age 18 to 80, and (iii) focusing on households rather than individuals.

First, from the model's perspective, the relevant notion of income used for measuring excess consumption is broad and includes transfers. This approach extends that of Catherine, Miller, and Sarin (2025), who incorporate Social Security benefits as a component of income, to include all sources of labor income and transfers available in our data. To that end, we measure income in the data as income from wages and salaries, the labor income component of proprietor's income, government transfers (unemployment benefits, Social Security, other government transfers), and private defined-benefit pension income. To obtain consistent data series, we reconcile differences in variable definitions among the various waves of the PSID (a non-trivial task, see Appendix A).

Second, we are interested in the entire life-cycle. To this end, we estimate the income distribution for a wide range of ages from 18 to 80.[7] Because our income concept includes transfers such as unemployment benefits and retirement income from public or private defined-benefit pension plans, we do not need to assume that households are working full-time. Instead, our estimation takes into account the full cross-section of sample households, including retirees, receipients of unemployment benefits, part-time employees, and so on.

Third, we focus on households, aggregating income across its adult members. This avoids the obligation to model demographic changes such as becoming married, divorced, or widowed. Instead, we simply follow households identified by the head of household in the data.

**Superstar Income Parameters.** To help the model match the level of wealth inequality in the high-interest rate regime (1980s), we follow Castaneda, Diaz-Gimenez, and Rios-Rull (2003) in enriching the income process in (A.1) – (A.3) with a superstar income state, which has a high income level $\eta_t^i = \eta^{sup}$. Households enter this state with probability $p_{12}^{sup}$ when they are in the regular income state, and return to a regular income state with probability $p_{21}^{sup}$ when they are in the superstar income state. The transition probability parameters $p_{12}^{sup} = 0.0002$ and $p_{21}^{sup} = 0.975$ are taken from Boar and Midrigan (2020), and imply a roughly 1% probability of entering in the superstar income state over one's lifetime. Conditional on entering, the state has an expected duration of 40 years. The income level $\eta^{sup}$ is then chosen to match the top-10% wealth share in 1983 exactly, which requires a value equal to 32.46 times average income.

[7]Since model households can survive to age 100, we assume that the age profile embedded in $\chi' X_t^i$ remains constant from age 80 onward.

**Income Process Discretization.** For the persistent component of the income process $\eta$, we use a nested structure, so that a household first draws whether it transitions into or out of the superstar state using the probabilities stated above. Conditional on ending out of the superstar state, the household then draws a new value of $\eta$ from the non-superstar distribution. For households beginning and ending in the non-superstar state, we approximate (A.2) with a discrete Markov chain $P_\eta$ using the method of Rouwenhorst (1995). Households transitioning from the superstar to non-superstar states draw a random non-superstar $\eta$ state from the ergodic distribution of $P_\eta$. Conditional on $\eta$, we draw i.i.d. values for $\nu$ using nodes and weights from Gaussian quadrature.

## 2.2 Stationary Economy: High Interest-Rate Regime

With the model calibrated, we now explore the quantitative properties of its stationary distribution under the initial high interest rate regime. Figure 1 displays the life cycle profiles of several key variables. The first two columns show income, consumption, financial wealth, and human wealth, with the axes are normalized such that 1 represents the median income during working life. Income displays the traditional hump-shape over the life cycle. Income inequality is increasing over the first half of the life cycle as income shocks accumulate. After retirement, households switch to our estimated over-65 income process which has lower dispersion. While this compresses income inequality beyond this point, we note that it remains non-negligible since agents have heterogeneous retirement income and still face some income risk after age 65.

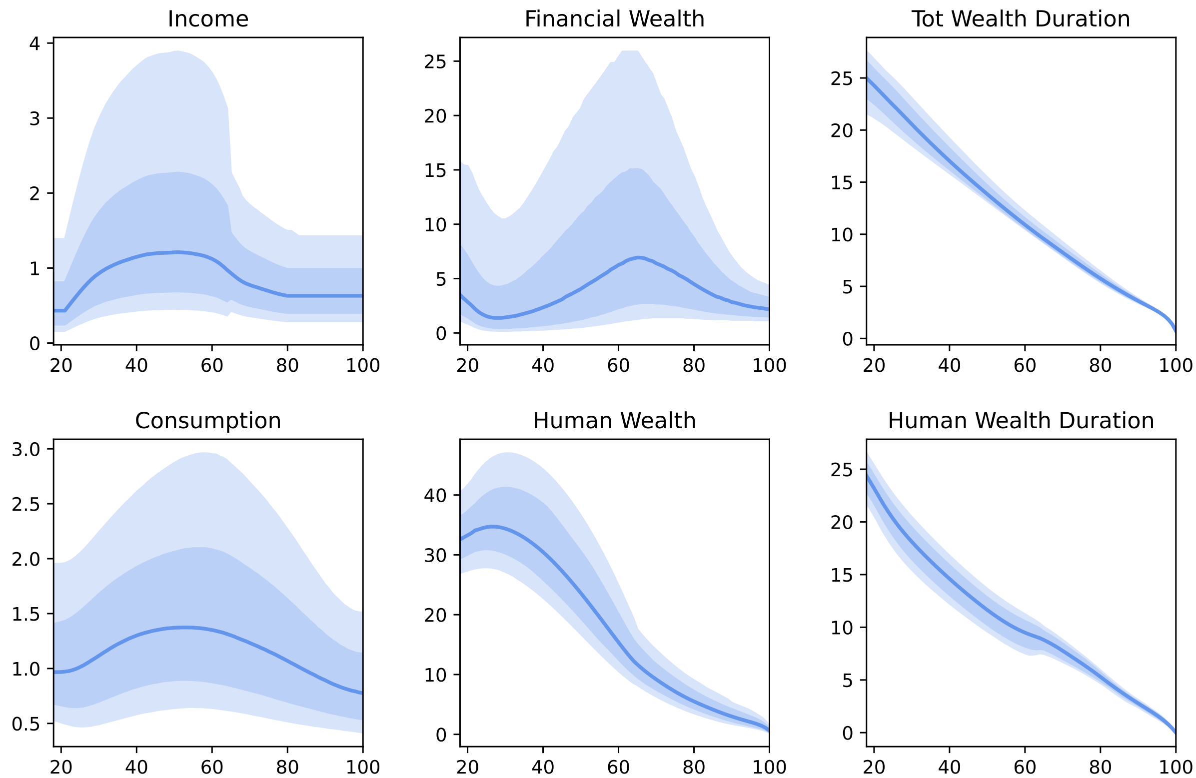
The bottom right panel shows that both the level and dispersion of consumption are rising over the working part of the life cycle, with dispersion falling in retirement when income risk declines. This is consistent with the data, which show that consumption inherits the hump-shaped profile from income (e.g., Krueger and Perri, 2006).

The top middle panel shows financial wealth, which increases in preparation for retirement, and is subsequently run down during retirement. Financial wealth inequality rises and falls over the life cycle. The first few years are influenced by bequests, which households receive at age 18. On average, households spend these down before beginning to save for retirement closer to age 30, while the large dispersion in the size of bequests generates the initial peak in financial wealth dispersion at the start of the life cycle.[8]

The bottom middle panel shows human wealth, defined as the present value of income. Human wealth is generally declining in age as there are fewer remaining periods of income remaining. However, human wealth rises in the early years of the life cycle as the households' highest-earning periods are brought closer to the present. Total wealth for young households consists almost exclusively of human wealth, except for a small share of households who received large bequests. As households age and prepare for retirement, a larger share of total wealth becomes

[8]To the extent that actual households receive bequests later in life, this would increase financial inequality between young and old, strengthening our effects.

Figure 1: Life Cycle Profiles




*Note:* This figure plots the life cycle profiles by age for the all agents of all groups combined. The axes are normalized so that the average income across all agents of all ages is equal to unity. The center line displays the median, while the dark and light bands represent 66.7% and 95% percentile bands.

financial wealth. However, human wealth remains the largest component of total wealth for most households until the typical retirement age.

Appendix Figure D1 displays the Lorenz curves for consumption and wealth for all households (in all groups), and reports the Gini coefficients. The model generates a Gini coefficient for (after-transfer) household income of. Consistent with the data, consumption inequality is somewhat lower than income inequality, and has a Gini coefficient of. Financial wealth is much more unequally distributed than human wealth or total wealth, with a Gini coefficient of  compared to  and, respectively. This much lower inequality in total wealth arises from a combination of (i) the importance of human wealth in total wealth, and (ii) the negative cross-sectional correlation between financial wealth, dominated by the middle aged and old, and human wealth, which is highest for the young.

The right panels of Figure 1 display the duration of human and total wealth by age. Human wealth represents a claim on lifetime income whereas total wealth represents a claim on lifetime consumption. Both of these durations are similar because of the importance of human wealth in total wealth. These durations are high when young, around 25, and drop rapidly as age increases, as there are fewer years of income remaining.

# 3 Results: The Compensated Wealth Distribution

Having answered our first question on the quantitative role of interest rates in driving financial wealth inequality, we now turn to our second question: what are the implications of this change for consumption and total wealth inequality?

To measure the effects of interest rate changes on consumption opportunities, we compare the evolution of the financial wealth distribution in our baseline economy compared to a counterfactual *compensated* wealth distribution in which, following each shock, households receive exactly enough wealth to be able afford their previous consumption plans. We compute this as

$$\tilde{\theta}^{comp} = \mathbb{E}_0 \left\{ \sum_{t=0}^{T-1} \tilde{R}^{-t}(c_t - y_t) + \tilde{R}^{-T}\theta_T \right\} \tag{17}$$

where $\{c_t\}$ and $\theta_T$ (bequest) make up the pre-shock consumption plan, and $\tilde{R}$ is the post-shock interest rate. From Proposition 1, we know that if a household ends up with this amount of wealth, it will exactly keep its pre-shock consumption plan in the zero-mortality limit, meaning that under the compensated wealth distribution household consumption is approximately unaffected by interest rate shocks. Thus, we can use deviations between the repriced and compensated wealth distribution to measure the real consumption effects of a change in rates, as distinct from paper gains and losses. Because this measurement depends only on the budget constraint, and not on the household utility function, we view it as robust compared to welfare calculations that could depend heavily on household preferences.

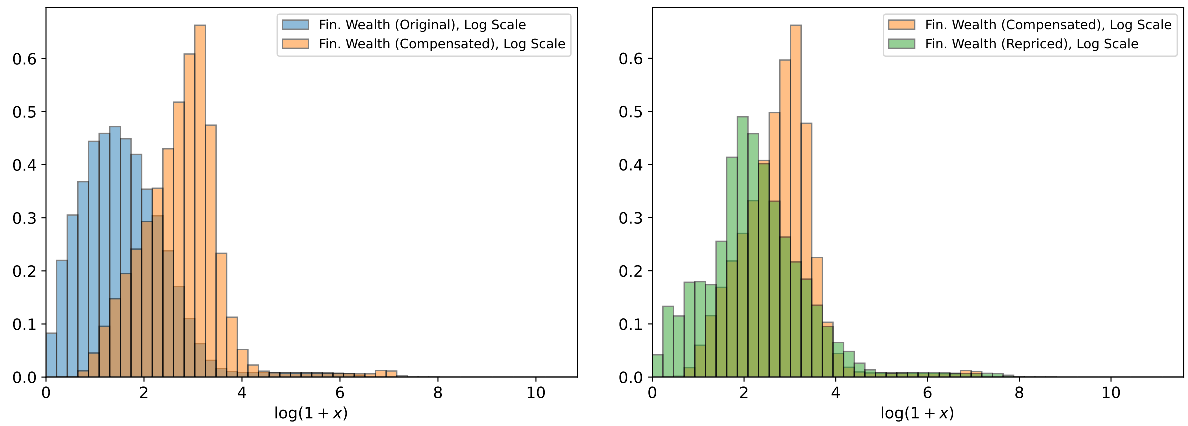
## 3.1 The Compensated Distribution: One-Shot Experiment

We begin our analysis of the compensated distribution in our one-shot experiment. Figure 2 presents the distribution of financial wealth following our one-time decline in interest rates, alongside the original (pre-shock) distribution, showing two major differences between the two.[9] First, the compensated distribution is shifted substantially to the right from the original wealth distribution as the present values of households' excess consumption plans increase under low rates. As a result, the aggregate amount of financial wealth in the compensated distribution exceeds the pre-shock total by%. As can be seen from the plot, this rightward shift extends up to the very top, implying that even the wealthiest individuals must be compensated with additional financial

[9] To ensure that the full distribution is visible, we display transformed variables $\log(1 + x)$ on the x-axis.

Figure 2: Histogram, Compensated vs. Original Financial Wealth Distribution

(a) Original vs. Compensated (b) Compensated vs. Repriced




*Note:* This plot displays the distribution of financial wealth under the stationary distribution and under the compensated distribution drawn from the stationary distribution of the economy. The x-axis displays a transformation $\log(1+x)$ of the original data. Each distribution is top coded at the top 0.1% of the pre-shock wealth distribution.

assets to attain their old consumption plans. Indeed, nearly one third (%) of new financial wealth accrues to top-1% financial wealth holders under the compensated distribution.

Second, although all households including the wealthiest see financial wealth increase under the compensated distribution, the less wealthy gain proportionally more, reducing financial wealth inequality. Visually, while the original high interest-rate distribution of financial wealth is heavily right-skewed, the compensated distribution with low rates is actually left-skewed. Quantitatively, the share of financial wealth held by the top-10% decreases from% in the baseline economy to% in the compensated economy.

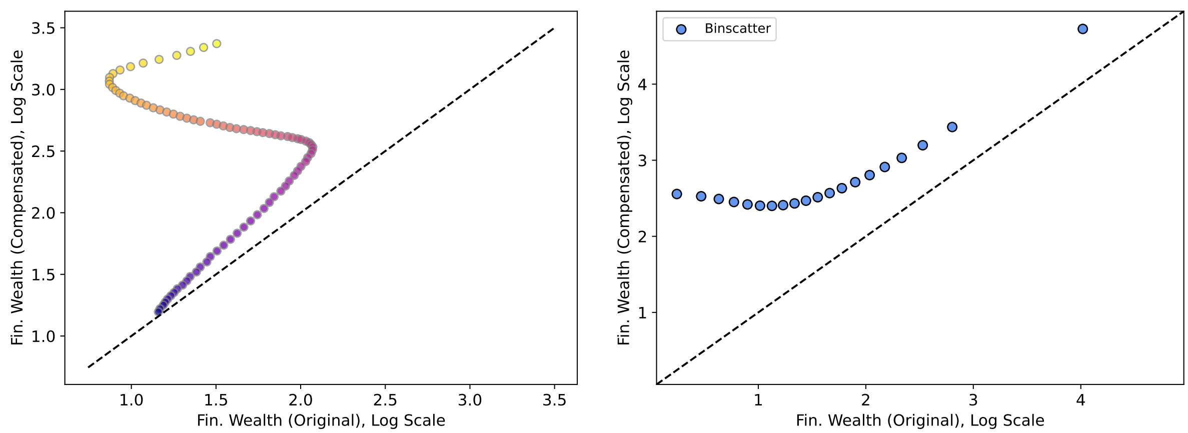
To see why inequality falls in the compensated distribution, we turn to Figure 3. Panel (a) compares medians by age under the original (horizontal axis) and compensated (vertical axis) financial wealth distributions by age. The youngest agents (light/yellow) have a low or intermediate level of wealth in the original distribution, but require the largest increase in financial wealth (vertical distance above the 45-degree line) in the compensated distribution. Households approaching retirement have more initial financial wealth, and require less compensation following the shock. Finally, the oldest households (dark/purple) have low wealth and require the least compensation.

This result may be surprising, since the young have the majority of their portfolio "invested" in human wealth, which has a long duration (bottom right panel of Figure 1), and thus provides a natural hedge against interest rate changes. However, the young plan to save during middle age ($c_t < y_t$), then dissave during retirement ($c_t > y_t$). This gives the young a very long duration of excess consumption $D^{c-y}$, making their original excess consumption plan much more expensive under low rates. In practical terms, the young will struggle to build retirement wealth and earn income on that wealth in retirement under low rates, making their pre-shock consumption plans

Figure 3: Scatterplots, Compensated vs. Original Financial Wealth Distribution

(a) Medians by Age

(b) Means by Fin. Wealth




*Note:* Panel (a) plots the distribution of original financial wealth against the distribution of compensated financial wealth by age. Each dot represents the population mean for one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Both variables are plotted using the transform $\log(1+x)$. The dashed line represents equality between the original and compensated distributions. Panel (b) plots the same distribution by bins of original financial wealth in place of age.

unattainable without large infusions of financial wealth today. In contrast, older agents have lower values of $D^{c-y}$ as retirement spending is brought closer to the present, and require less compensation. These households have already benefited from the higher rate of return in accumulating their retirement assets, while the oldest are already dissaving, consuming principal rather than interest, making them less affected by the loss of high-return investment opportunities.

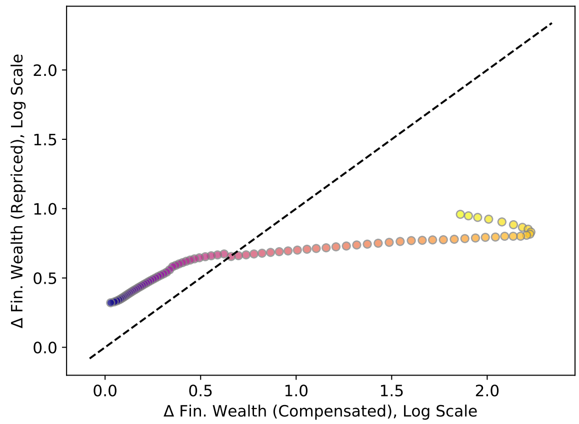
Panel (b) of Figure 3 aggregates over ages to present the total compensation required for various levels of pre-shock financial wealth. The lowest levels of financial wealth mix young agents who have not begun saving with old agents who are spending down assets late in life. As a result, the low wealth group mixes over agents requiring the largest and smallest amounts of compensation. Quantitatively, the young make up a larger share of this group and dominate the aggregate result, so that the least wealthy agents in this economy require the most compensation. Higher wealth levels contain an increasing share of middle-aged individuals, who require non-zero levels of compensation, but less than those at the bottom of the wealth distribution.

To separate the influence of age and wealth, Panel (a) of Appendix Figure D3 reproduces Panel (b) controlling for age fixed effects. It shows that wealthier households also require less compensation even after controlling for age.

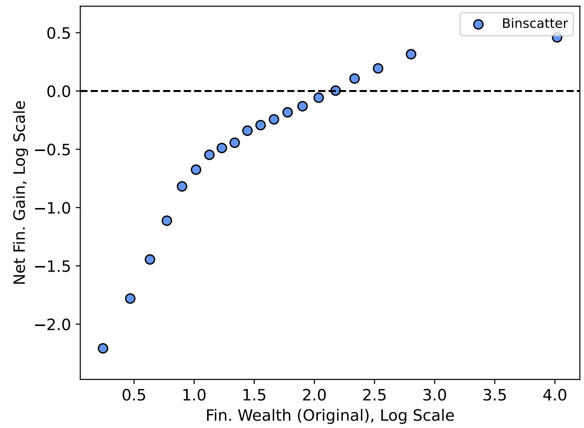
To summarize, we observe a negative relationship between initial wealth and required compensation, due both to the life-cycle pattern of wealth, and to the links between wealth and required compensation independent of age. This negative relationship means that wealthier house-

Figure 4: Scatterplots, Repriced Financial Wealth Distribution

(a) Compensated vs. Repriced: Age Medians




(b) Scatter: Repriced Gain




*Note:* This plot displays the distribution of financial wealth under the repriced distribution, compared to the compensated distribution. Panel (a) displays the change in financial wealth relative to the original distribution for the compensated (x-axis) and repriced (y-axis) distributions. Both axes display a transformation $\log(1+x)$ of the original data. Each dot represents one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Panel (b) displays original financial wealth on the x-axis and the net financial gain (repriced minus compensated wealth) on the y-axis. The x-axis displays the transform $\log(1+x)$, while the y-axis displays the difference in transformed values. Each dot represents 5% of households from the original wealth distribution. All distributions are drawn from the stationary distribution of the economy.

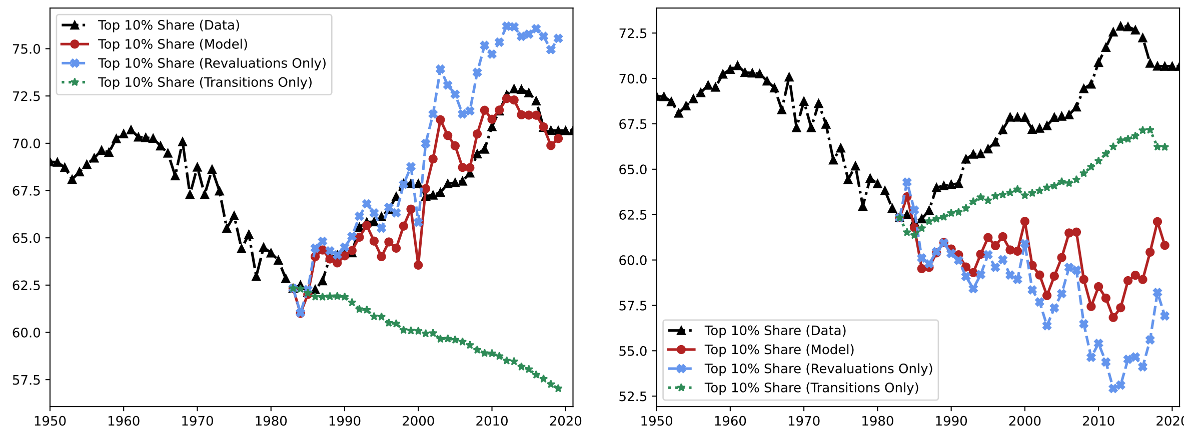
holds need to gain the least under the compensated distribution, leading to a decrease in inequality in this counterfactual environment.

**Comparison: Compensated vs. Repriced Distributions** Having computed the compensated financial wealth distribution required to keep consumption plans constant, we can compare it to the repriced financial wealth distributions actually observed under low interest rates. Panel (b) of Figure 2 contrasts repriced and compensated distributions from our one-shot experiments. Since lower interest rates increase aggregate financial wealth by% by slightly more than the% increase in aggregate financial wealth required under the compensated distribution, there is more than enough wealth gain from repricing in aggregate to compensate all households. However, the compensated and repriced distributions display strikingly different shapes, with many more agents at low wealth levels in the repriced distribution compared to the compensated distribution.

To zoom in on the winners and losers from lower rates, Figure 4 compares changes in the repriced vs. compensated distributions by age in Panel (a) and by wealth in Panel (b). Panel (a) shows that while repricing delivers some gains to the young, it does not satisfy their large need for compensating transfers, leaving them well below the 45-degree line. In contrast, the middle-aged and old are over-hedged, receiving more wealth under repricing than needed to afford their

Figure 5: Top-10% Share, Gradual Transition Exercise

(a) Top-10% Share: Repriced (b) Top-10% Share: Compensated




*Note:* The left panel plots the top-10% financial wealth share in the data (black dash-dotted line) and in the model with repricing (red solid line). It decomposes the evolution in the top-10% financial wealth share into a component solely due to instantaneous repricing (blue line, $\hat{S}_t^{10,REV}$) and component due to optimal consumption-savings decisions (green line, $\hat{S}_t^{10,MR}$). The right panel provides the same information as the left panel except for the compensated wealth distribution.

former consumption plan, as shown by their position above the 45-degree line. Thus, the young will see their consumption possibilities contract, while older households will see them expand.

Panel (b) of Figure 4 displays the net gain from repricing, defined as the change in repriced wealth net of the change in compensated wealth. The figure reinforces our previous finding, showing that wealthier households gain on net from repricing, while the least wealthy experience a large net loss from the interest rate change, as repricing fails to appropriately compensate these households. Panel (b) of Appendix Figure D3 shows that the same pattern holds to a lesser degree across the wealth distribution after controlling for age fixed effects.

### 3.2 The Compensated Distribution: Transition Experiment

Our main quantitative experiment feeds in a gradual sequence of interest rate changes. The red line in Panel (b) of Figure 5 shows how the top-10% wealth share would have evolved under the compensated wealth distribution — the path of inequality that would have been required to keep consumption plans constant. The last column of Table 1 summarizes these changes. The compensated wealth distribution sees a substantial reduction in financial wealth inequality. The top-10% financial wealth share falls by -1.5pp between 1983 and 2019 and the Gini falls by -0.020 , while the top-1% share rises modestly by +0.6pp. The effect is largely captured by the instantaneous compensation following each interest rate change (blue line in Panel (b) of Figure 5), somewhat offset by the effect coming from consumption-savings choices (green line).

Table 1: Change in Inequality, Gradual Transition Experiment

| | Data (WID) | Repriced | Compensated |
|---|---|---|---|
| Top-10% FW | +8.3pp | +7.9pp | -1.5pp |
| Top-1% FW | +11.3pp | +6.4pp | +0.6pp |
| Gini FW | +0.054 | +0.061 | -0.020 |
| Top-10% HW | – | +1.1pp | +1.1pp |
| Top-1% HW | – | -2.2pp | -2.2pp |
| Gini HW | – | +0.066 | +0.066 |
| Top-10% TW | – | +0.6pp | -1.6pp |
| Top-1% TW | – | -1.9pp | -2.7pp |
| Gini TW | – | +0.059 | +0.039 |

*Note:* The table reports the change in the Top-10% share, Top-1% share, and Gini Coefficient of financial wealth (FW, top panel), human wealth (HW, middle panel), and total wealth (TW, bottom panel). The change is measured between 1983 and 2019 in the model (Repriced and Compensated columns) as well as in the Data (WID) column.

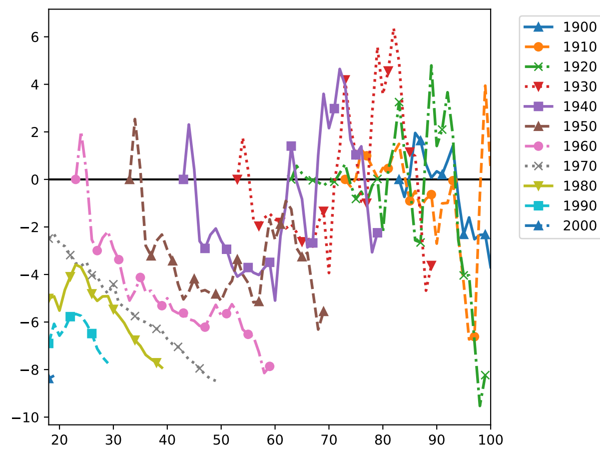
In sum, the starkly diverging paths for financial wealth inequality in the repriced and compensated distributions imply that changing interest rates do not merely result in paper gains but result in important changes in consumption possibilities. The young and poor see substantial deterioration in their consumption because the duration of their financial wealth is below the duration of their excess consumption. The opposite is true for the old and the rich, whose consumption opportunities expand from declining rates.

Last, we can evaluate changes under the compensated distribution in total wealth inequality, which summarizes household lifetime consumption. Table 1 shows that total wealth inequality increases by less (or decreases by more) under the compensated distribution compared to the repriced distribution. Thus, our conclusions regarding the real effects of falling rates on inequality carry over from financial wealth to total wealth. Quantitatively, however, the gaps between repriced and compensated inequality measures are substantially smaller for total than for financial wealth. This reflects the influence of human wealth, which represents the majority of total wealth, and undergoes identical changes in the compensated and repriced distributions.

## 4 Cohort-Level Analysis

For our final set of results, we use the model to shed light on how different cohorts have gained and lost over our sample period. Recall that a decline in rates generally harms households when they are younger ($D^\theta < D^{c-y}$) and benefits them when they are older ($D^\theta > D^{c-y}$). Thus, to determine the total impact of falling rates on a given household, it matters not only how much rates fell during its lifetime, but also exactly when during its life cycle this occurred.

Figure 6: Cohort Total Wealth Outcomes, Transition Experiment




*Note:* The graph plots total wealth, the present discounted value of consumption, along the observed interest rate path for the median member of a birth cohort, indicated by the various colored lines, in percent deviation of what that wealth path would have been in the steady state with high (1983) interest rates. Each cohort is labeled with the first year of the relevant birth decade, for example "1900" represents households born between 1900 and 1909.

To address this, Figure 6 plots total wealth (present value of lifetime consumption) for the median member of each birth-decade cohort at each age in their lives. We normalize the series as percent deviations in median total wealth compared to households of the same age in the initial 1983 stationary distribution. These series thus represent the effect of the fall in rates on the lifetime consumption value of each cohort compared to a world in which rates remained unchanged. Each line represents a cohort of ten birth years, with the oldest cohort (born 1900–1909) well into retirement at the start of our sample, and the youngest cohort (born 2000–2009) entering the workforce only in the sample's final year.

The graph shows that the older cohorts gain while the younger cohorts lose. Households born before 1920 observe only modest gains since they have mostly run down their wealth by the time the interest rate declines begin. Households born in the 1920s through 1940s are the biggest winners, experiencing peak total wealth gains in excess of 5%. The youngest cohorts (Gen X, Millennials, and Gen Z) strictly lose, experiencing peak total wealth losses approaching 8% compared to households of the same age in the stationary economy.

## 5 Conclusion

A persistent decline in real interest rates, like the one experienced in much of the world between the 1980s and the 2010s, leads to a rise in financial wealth inequality when there is a positive covariance between financial wealth levels and the duration of financial wealth across households.

Using detailed portfolio data, we show that this condition is met in the U.S. data, and that the duration heterogeneity is large enough to account for the entire rise in the top-10% share of financial wealth. With the help of a standard consumption-savings model, we show that the reduction in interest rates not only leads to "paper valuation gains" but affects consumption possibilities. In particular, young and less wealthy households are forced to save at lower rates for their retirement by purchasing more expensive assets in the future. They see their consumption possibilities contract when rates fall. Older and wealthier households have more than enough duration in their portfolio to allow them to afford the old consumption plan under the new, lower interest rates, thanks to large capital gains. We show how these effects played out in the data by studying how different cohorts' consumption possibilities were affected by the observed path of interest rates.

Recently, long-term real rates have begun to rise after a 40-year decline. Between March 2022 and August 2023, the 10-year real bond yield increased 2.65% points. Our paper predicts that this sharp rise in real rates will lower financial wealth inequality and benefit the consumption opportunities of the young and the poor in the years to come.

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# Online Appendix

## A Labor Income Process

### A.1 Main Specification

The labor income process consists of a regular component and a superstar component. The regular income process for household $i$ of age $a$ at time $t$ that is not currently in the superstar state takes the form, standard in the literature, given by:

$$\log\left(y^i_{t,a}\right) = m_t + \chi' X^i_t + z^i_t, \tag{A.1}$$

$$z^i_{t+1} = \alpha_i + \eta^i_{t+1} + \nu^i_{t+1}, \tag{A.2}$$

$$\eta^i_{t+1} = \rho\eta^i_t + u^i_{t+1}, \tag{A.3}$$

where $m_t$ is a year-fixed effect and $X^i_t$ is a vector of household characteristics that includes a cubic function of age.[10] We normalize the mean of the age profile to unity during working life. The stochastic income component $z^i_t$ contains a household-fixed effect $\alpha^i$, a persistent component $\eta^i_{t+1}$, and an i.i.d. component $\nu^i_{t+1}$. We set $\mathbb{E}[\nu^i] = E[u^i] = \mathbb{E}[\alpha^i] = \mathbb{E}[\eta^i_0] = 0$, while $Var[\nu^i] = \sigma^2_\nu$, $Var[u^i] = \sigma^2_u$, $Var[\alpha^i] = \sigma^2_\alpha$, and $Var[\eta^i_0] = \sigma^2_{\eta,0}$.

To allow for lower income risk during retirement, we re-estimate (A.1)–(A.3) separately for households above and below age 65, and assume that households face income risk that switches when they turn 65. The parameters are estimated by GMM using PSID data from 1970 until 2017, as detailed in Appendix A.3. Figure A1 below plots the deterministic life-cycle income profile.

We enrich the regular income process in (A.1)–(A.3) with a superstar income state following Castaneda, Diaz-Gimenez, and Rios-Rull (2003). This state has a high income level $\eta^i_t = \eta^{sup}$; households enter the state with probability $p^{sup}_{12}$, and exit with probability $p^{sup}_{21}$. The values $p^{sup}_{12} = 0.0002$ and $p^{sup}_{21} = 0.975$ are taken from Boar and Midrigan (2020), and imply a roughly 1% probability of entering in the superstar income state over one's lifetime. Conditional on entering, the state has an expected duration of 40 years. We calibrate the superstar income level $\eta^{sup}$ so that the top-10% wealth share in the model's stationary economy exactly matches the average top-10% wealth share in the WID data over our 1983-2023 sample. This requires a value equal to 32.46 times average income.

A household first draws whether it transitions into or out of the superstar state using the probabilities stated above. Conditional on being in the regular income state, the household then draws a new value of $\eta$. For households beginning and ending the period in the non-superstar

[10] Our results are similar if we estimate the year fixed effect and the age profile separately for groups of households that depend on education (college completion or not), race (white or non-white), and gender of head of household (8 groups total). Since it makes little difference for our main results, we assume ex-ante identical households for simplicity.

state, we approximate (A.2) with a discrete Markov chain $P_\eta$ using the method of Rouwenhorst (1995). Households exiting the superstar state draw a random $\eta$ from the ergodic distribution of $P_\eta$. Conditional on $\eta$, we draw i.i.d. values for $\nu$ using Gaussian quadrature.

## A.2 Data Source: PSID

The Panel Study of Income Dynamics (PSID) is a household panel survey that began in 1968. The PSID was originally designed to study the dynamics of income and poverty. Thus, the original 1968 PSID sample was drawn from two independent samples: a sample of 1,872 low income households from the Survey of Economic Opportunity (the "SEO sample") and a nationally representative sample of 2,930 households designed by the Survey Research Center at the University of Michigan (the "SRC sample"). In this paper, we use the "SRC sample" for the time period from 1970 until 2017.

### A.2.1 PSID Income variables

We now describe the construction of the relevant income variables used in the paper. We construct the following variables: *labinc2f* is labor income excluding transfers but including the labor part of business and farm income for both head and eventual spouse; *transf* which are total households transfer (including Social Security Income and other transfers); *labinc3f*, which is our measure of total household income for both head and eventual spouse, is the sum of *labinc2f* and *transf*.

We provide further details on how we build these three variables. As the variables included in the PSID are subject to change, the variable construction vary with different sample period. For this reason, below we provide details on the variables used in different time periods. Moreover, the ticker for each variable changed in each survey. We therefore define the ticker used in a specific year as (YYYY:Ticker).[11]

**labinc2f** In the 1970 - 1993 sample, this variable is defined as the sum of Total labor income of head, including wages and salaries, labor part of business income and farm income (1993:V23323), and Spouse's total labor income, including labor part of business income and farm income (1993:V23324). In the 1993 - 2017 sample, this variable is defined as the sum of Reference Person's total labor (including wages and other labor) excluding Farm and Unincorporated Business Income, (2017:ER71293), Labor Part of Business Income from Unincorporated Businesses (2017:ER71274), Reference Person's and Spouse's/Partner's Income from Farming (2017:ER71272), Wife's Labor Income, Excluding Farm and Unincorporated Business Income (2017:ER71321), Wife's Labor Part of Business Income from Unincorporated Businesses (2017:ER71302). Note that farm's income includes both labor and asset portions of income.

[11] The PSID website provides information on how to harmonize tickers across different surveys.

**transf** In the 1970-1993 sample, this variable is defined as Total Transfer Income of Head and Wife/"Wife" (1993:V22366) and Total Transfer Income of Others (1993:V22397). In the 1994-2003 sample, this variable is defined as Head's and Wife's Total Transfer Income, Except Social Security (2017:ER71391), Other Total Transfer Income, Except Social Security (2017:ER71419), Total Family Income from Social Security (1994:ER4152). In the 2004-2017 sample, this variable is defined as: Head's and Wife's Total Transfer Income, Except Social Security (2017:ER71391), Other Total Transfer Income, Except Social Security (2017:ER71419), Reference Person's Income from Social Security (2017:ER71420), Spouse's/Partner's Income from Social Security (2017:ER71422), Others Income from Social Security (2017:ER71424).

**labinc3f** We then construct *labinc3f* by summing total family labor income (*labinc2f*) and total family transfers (*transf*).[12]

### A.3 Estimating the Income Process

**Age Profile** We estimate the age profile of income following Deaton and Paxson (1994). First, we estimate the average income for each cohort in each year, using PSID data. $y_{c,t}$ is the average income of cohort $c$ at time $t$, based on our *labinc3f* definition of income. Then, we estimate the following regression model:

$$\log y_{c,t} = \beta + \gamma_a + \gamma_c + \gamma_t + \varepsilon_{c,t}, \tag{A.4}$$

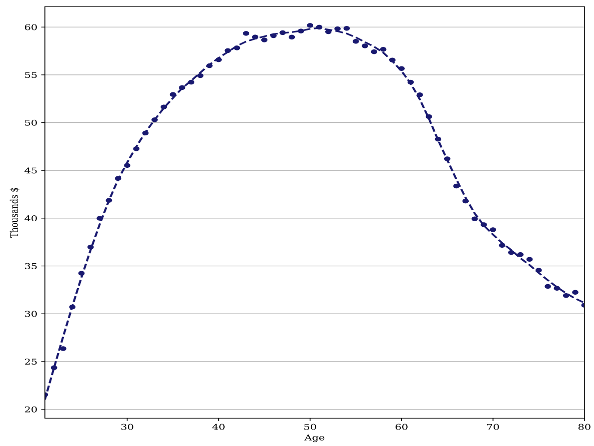
where the subscript $c, t, a$ refers to cohort, time and age, respectively. We define as $c$ the age at time $t = 0$ (i.e. 1970). Due to the linear relationship between age, cohort and time, we cannot separately identify the different fixed effects. We hence resort to the method used by Deaton and Paxson (1994): we attribute growth to age and cohort effects, while we use the year effects to capture cyclical fluctuations or business-cycle effects that average to zero over the long run. We hence constraint the year fixed effects to be orthogonal to a time trend and to sum to zero. We then estimate Equation A.4 using constrained OLS.

Figure A1 plots the estimates for the age dummies. The dots are based on the estimated dummies. The dashed lines apply a Savitzky–Golay filter to smooth the estimates and characterize our deterministic age-profile.

**Income Risk** In the second stage, we estimate income risk. We estimate (A.1) year by year and include cubic function of age as well as a set of fixed-effects: education, race, gender, state. We then extract the residuals $z_{it}$. Finally we estimate the risk parameters by GMM as detailed below.

[12] We have verified that aggregating the PSID results in a series that is close to the NIPA series.

Figure A1: Income Profile




*Note:* This figure displays the expected income profile evaluated at the 2016 year-fixed effects. The graph plots the expected income profile for the average person who is 21 years old in 2016, expressed in thousands of 2016 dollars. The model is estimated according to Equation (A.4) on PSID data from 1970 to 2017.

Using Equation (A.1)-(A.3), and define $j$ as equal to the age of the households minus the minimum age (21), we find that:

$$
\begin{aligned}
E[z^i_j, z^i_{j+h}] &= \sigma^2_\alpha + E[{\varepsilon^i_j}^2] + \sigma^2_\nu \qquad \text{if } h = 0, \\
E[z^i_j, z^i_{j+h}] &= \sigma^2_\alpha + \rho^h E[{\eta^i_j}^2] \qquad \text{if } h > 0, \\
E[{\eta^i_j}^2] &= \rho^{2j}\sigma^2_{\eta_0} + \sum_{k=1}^{j} \rho^{2(j-k)}\sigma^2_u.
\end{aligned}
$$

We allow the variance to differ in working age ($w$) and retirement age ($r$), where the retirement age starts at 65. We fixed the variance of initial persistent shocks $\sigma^2_{\eta_0} = 0$, then use a GMM estimation to estimate $\theta = (\rho, \sigma_{v,w}, \sigma_{u,r}, \sigma_{v,r}, \sigma_{u,r}, \sigma_\alpha)$. We use a Minimum Distance Estimator, where the weighting matrix is the identity matrix. We only include sample moments estimated on 100 or more observations.

**Sample Selection.** We use PSID data from 1970 to 2017. As discussed in Heathcote, Perri, and Violante (2010), after survey year 1997, the data frequency goes from annual to biannual. To make the estimation consistent, in the first part of the sample 1970-1997 we also sample data at biannual frequency. We only include households whose head is 21 to 80 years old. We only include households which were in the survey for three or more periods. We exclude households with zero or negative income. In each year, we trim the top 2.5% of households by their income.

The point estimates are displayed in Table A1. These are the parameters used in the main text.

Table A1: Idiosyncratic Risk Parameter Estimates

| | $\sigma^2_\alpha$ | $\sigma^2_{v,w}$ | $\sigma^2_{u,w}$ | $\sigma^2_{v,r}$ | $\sigma^2_{u,r}$ | $\rho$ |
|---|---|---|---|---|---|---|
| Estimated Parameters | 0.0762 | 0.1605 | 0.0413 | 0.0906 | 0.0255 | 0.9152 |

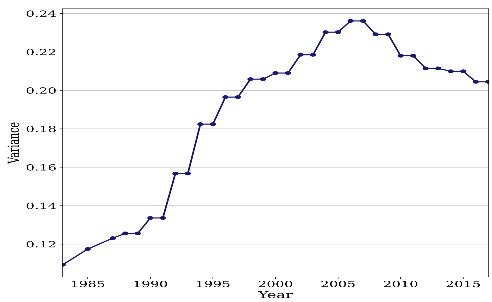
*Note:* $\theta = (\rho, \sigma_{v,w}, \sigma_{u,r}, \sigma_{v,r}, \sigma_{u,r}, \sigma_\alpha)$, are estimated using Equation (A.1)-(A.3); $\sigma^2_{\varepsilon_0}$ is fixed equal to 0. Data are based on PSID and runs from 1970 to 2017.

### A.3.1 Time Varying Income Risk

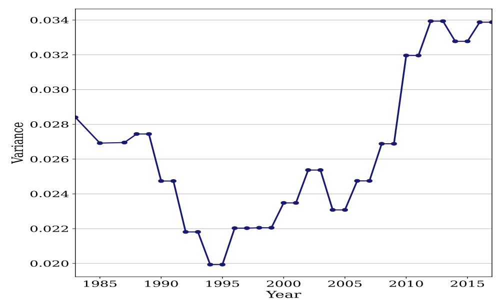
We also estimate the risk parameters using rolling sample of the PSID from 1983 till 2016. We use sample of 15 years (apart from 1983 and 1984 where we include data from 1970 to 1983 and 1984, respectively). We fix the autocorrelation $\rho$ of persistent shocks to the full sample value estimated in Table A1. Figure A2 plots time varying estimates of $\sigma^2_{u,w}$, $\sigma^2_{v,w}$, $\sigma^2_{u,r}$, $\sigma^2_{v,r}$ and $\sigma^2_\alpha$.

Figure A2: Time Varying Income Risk

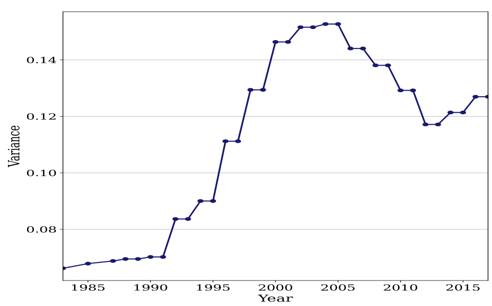
(a) Transitory Shocks - Working Age



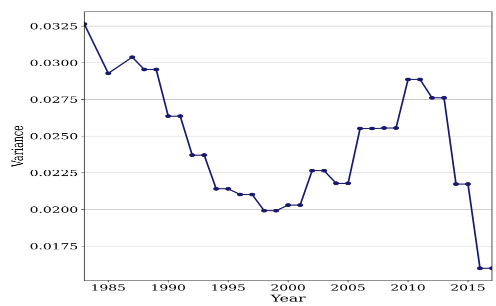

(b) Persistent Shocks - Working Age



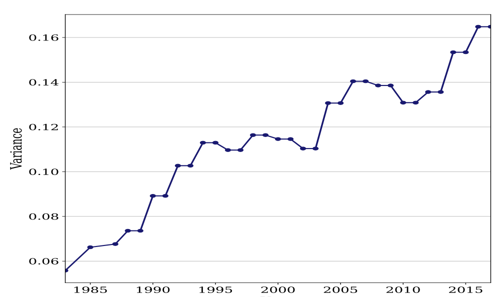

(c) Transitory Shocks - Retirement




(d) Persistent Shocks - Retirement




(e) Fixed-Effect Shocks




*Note:* This figure displays the the risk parameters using rolling sample of the PSID. We use sample of 15 years (apart from 1983 and 1984 where we include data from 1970 to 1983 and 1984, respectively). We fix the autocorrelation $\rho$ of persistent shocks to the full sample value estimated in Table A1. Panel A2a plots time varying estimates of $\sigma^2_{u,w}$, Panel A2b plots time varying estimates of $\sigma^2_{v,w}$, Panel A2c plots time varying estimates of $\sigma^2_{u,r}$, A2d plots time varying estimates of $\sigma^2_{v,r}$ and A2e plots time varying estimates of $\sigma^2_{\alpha}$. The model is estimated according to Equation (A.1)-(A.3) on PSID data from 1970 to 2017.

# B Proofs

## B.1 Proof of Proposition 1

We begin by proving (a). First, consider any consumption plan $\{c_t\}$ that exactly satisfies the lifetime budget constraint (10), which is without loss of generality due to local non-satiation. Similarly, let $\{\tilde{c}_t\}$ be a consumption plan that exactly satisfies the post-shock lifetime budget constraint

$$\theta_0 = \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t} (\tilde{c}_t - y_t) \right\} \tag{B.1}$$

Approximating the left hand side of (10) following a change in rates using (3) delivers

$$\log \tilde{\theta}_0 \simeq \log \theta_0 - D^{\theta} \times \varepsilon$$

which can be rearranged to yield

$$\log \theta_0 \simeq \log \tilde{\theta}_0 + D^{\theta} \times \varepsilon. \tag{B.2}$$

Similarly, we can approximate the right hand side of (10) to obtain

$$\log \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t} (c_t - y_t) \right\} \simeq \log \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} R^{-t} (c_t - y_t) \right\} - D^{c-y} \times \varepsilon$$

which can be rearranged to yield

$$\log \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} R^{-t} (c_t - y_t) \right\} \simeq \log \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t} (c_t - y_t) \right\} + D^{c-y} \times \varepsilon. \tag{B.3}$$

Substituting (B.2) and (B.3) into (10) and rearranging, we obtain

$$\log \tilde{\theta}_0 = \log \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t} (c_t - y_t) \right\} + (D^{c-y} - D^{\theta}) \times \varepsilon.$$

Finally, applying (B.1) to the left hand side delivers

$$\log \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t} (\tilde{c}_t - y_t) \right\} \simeq \log \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t} (c_t - y_t) \right\} + (D^{c-y} - D^{\theta}) \times \varepsilon \tag{B.4}$$

Assume that $\varepsilon < 0$. Examining (B.1) shows that the pre-shock consumption plan $\{c_t\}$ is affordable under the post-shock budget constraint if and only if

$$\mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t}(c_t - y_t) \right\} \leq \mathbb{E}_0 \left\{ \sum_{t=0}^{\infty} \tilde{R}^{-t}(\tilde{c}_t - y_t) \right\}. \tag{B.5}$$

From equation (B.4), we know this is true if and only if $D^\theta \geq D^{c-y}$. If $D^\theta > D^{c-y}$, then we have that (B.5) holds strictly. In this case, the household is able to afford a new consumption plan $\{\tilde{c}_t\}$ that is at least as large as $(\{c_t\}, b_T)$ in histories, and is strictly larger after some histories. As a result, this household's consumption possibilities expand. A symmetric argument shows that if $D^\theta < D^{c-y}$, then the household can no longer afford its pre-shock consumption plan following the fall in rates, implying that its consumption opportunities contract. Last, if $D^\theta = D^{c-y}$ the household's consumption opportunities are unchanged, meaning that a post-shock consumption plan is affordable under (B.1) if and only if it was also affordable prior to the shock under (10). We note that a symmetric proof using (9) in place of (10) would prove part (a) of the proposition even in the case where mortality risk is nonzero.

Part (b) follows directly from the optimality condition (8), which as mortality risk goes to zero takes the familiar form

$$c^{-\gamma} = \beta R \mathbb{E}\left[ (c')^{-\gamma} \,\middle|\, z \right]. \tag{B.6}$$

Since $\beta R = \tilde{\beta}\tilde{R}$, any consumption plan that satisfied (8) will still satisfy (B.6) under $\tilde{R}$ and $\tilde{\beta}$. Since the budget constraint (B.1) is satisfied by assumption, this completes the proof of part (b).

Part (c) follows directly from parts (a) and (b). From part (a) we know that if $D^\theta = D^{c-y}$ then the household will have exactly enough financial wealth post-shock to afford its pre-shock consumption plan, while part (b) implies that it will still find this plan optimal.

Alternatively, assume that $D^\theta \neq D^{c-y}$. If $D^\theta < D^{c-y}$ we know from part (a) that the original consumption plan is no longer affordable, and cannot be an equilibrium selection of the household. If $D^\theta > D^{c-y}$ then the household's consumption opportunity set has strictly expanded. In this case, the household should pick a sequence with $\tilde{c}_t > c_t$ in some states due to local non-satiation. Thus, if $D^\theta \neq D^{c-y}$ the pre-shock consumption plan is either infeasible or suboptimal.

Combined, we have shown that a household's consumption plan is unchanged following the shock if and only if $D^\theta = D^{c-y}$.

# C Incomplete Markets Model with Aggregate Risk

This appendix sets up an infinite-horizon model where ex-ante identical households face both idiosyncratic and aggregate income risk. Interest rates are determined in equilibrium. It first shows how to map the model with stochastic growth into a stationary model without aggregate risk in the spirit of Bewley (1986). We define an equilibrium under high interest rates. We show how to compute the value of human wealth in a matter consistent with the aggregate resource constraint. The main results are in Section C.5. They characterize how a decline in rates affects wealth inequality.

## C.1 Model Setup

### C.1.1 Endowments

Time is discrete, infinite, and indexed by $t \in [0, 1, 2, ...)$. The aggregate endowment $e$ follows the stochastic process:

$$e_t(z^t) = e_{t-1}(z^{t-1})\lambda_t(z_t)$$

where $\lambda(z_t)$ denotes the stochastic growth rate of the aggregate endowment and $z_t$ the aggregate state. The history of aggregate shocks is denoted by $z^t = \{z_t, z_{t-1}, \cdots\}$. A share $\alpha_t(z_t)$ of the aggregate endowment is financial income (dividends), the remaining $1 - \alpha_t(z_t)$ share represents aggregate labor income.

Households are subject to idiosyncratic income shocks, whose history is denoted by $\eta^h = \{\eta_h, \eta_{h-1}, \cdots\}$. The $\eta_h$ shocks are i.i.d. across households and persistent over time. The idiosyncratic shock process is assumed to be independent from the aggregate shock process. Labor income $y$ follows the following stochastic process:

$$y_t(z^t, \eta^h) = \widehat{y}_t(z^t, \eta^h)(1 - \alpha_t(z_t))e_t(z^t),$$

The ratio of individual to aggregate labor income, which we refer to as the labor income share, is given by $\widehat{y}_t(z^t, \eta^h)$. We use $(z^t, \eta^h)$ to summarize the history of aggregate and idiosyncratic shocks, and $\pi(z^t, \eta^h)$ to denote the unconditional probability that state $s^t$ will be realized. If the aggregate and idiosyncratic states are independently distributed, then we can decompose state transition probabilities into an aggregate and idiosyncratic component:

$$\pi(z_{t+1}, \eta_{h+1}|z^t, \eta^h) = \phi(z_{t+1}|z^t)\varphi(\eta_{h+1}|\eta^h),$$

We make this assumption of independence between aggregate and idiosyncratic labor income risk in what follows.

#### C.1.2 Preferences

A household maximizes discounted expected utility:

$$U(c) = \sum_{j=1}^{\infty} \beta^j \sum_{(z^{t+j},\eta^j)} \phi(z^{t+j})\varphi(\eta^j)\frac{c(z^{t+j},\eta^j)^{1-\gamma}}{1-\gamma},$$

where the coefficient of relative risk aversion $\gamma > 1$, and the subjective time discount factor $0 < \beta < 1$.

#### C.1.3 Technology

Households choose a portfolio of state-contingent bonds $a_t(z^t, \eta^h; z_{t+1})$ for each state $z_{t+1}$, which trade at prices $q_t(z^t, z_{t+1})$, and shares in the Lucas tree (stocks) $\sigma_t(z^t, \eta^h)$, which trade at price $\nu_t(z^t)$ satisfying the budget constraint:

$$c_t(z^t, \eta^h) + \sum_{z_{t+1}} a_t(z^t \eta^h; z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(z^t, \eta^h)\nu_t(z^t) \leq W_t(z^t, \eta^h).$$

Household cash on hand $W$ evolves according to:

$$\begin{aligned} W_{t+1}(z^{t+1}, \eta^{h+1}) &= a_t(z^t \eta^h; z_{t+1}) + \hat{y}_{t+1}(z^{t+1}, \eta^{h+1})(1-\alpha(z_{t+1}))e_{t+1}(z^{t+1}) \\ &+ \left(\alpha(z_{t+1})e_{t+1}(z^{t+1}) + \nu_{t+1}(z^{t+1})\right)\sigma_t(z^t, \eta^h). \end{aligned}$$

Households are subject to state-uncontingent and state-contingent borrowing constraints:

$$\sum_{z_{t+1}} a_t(z^t \eta^h; z_{t+1}) q_t(z^t, z_{t+1}) + \sigma_t(s^t)\nu_t(z^t) \geq K_t(s^t)$$

$$a_t(z^t \eta^h; z_{t+1}) + \left(\alpha(z_{t+1})e_{t+1}(z^{t+1}) + \nu_{t+1}(z^{t+1})\right)\sigma_t(s^t) \geq M_t(s^t, z_{t+1})$$

where $K$ and $M$ denote generic borrowing limits. Incomplete risk sharing arises from two sources: the lack of an asset whose payoff depends on the idiosyncratic income shock $\eta^t$ and the borrowing constraints.

### C.2 Transformation into Stationary Economy

We can transform the stochastically growing economy into a stationary economy with a constant aggregate endowment following Alvarez and Jermann (2001); Krueger and Lustig (2010). To that end, define the stationary consumption allocations:

$$\hat{c}_t(z^t, \eta^h) = \frac{c_t(z^t, \eta^h)}{e_t(z^t, \eta^h)}, \forall (z^t, \eta^h),$$

the stationary transition probabilities and the stationary subjective time discount factor:

$$\widehat{\phi}(z_{t+1}|z^t) = \frac{\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}}{\sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}},$$
$$\widehat{\beta}(z^t) = \beta\sum_{z_{t+1}}\phi(z_{t+1}|z^t)\lambda_{t+1}(z_{t+1})^{1-\gamma}.$$

Agents in the stationary economy with these preferences:

$$U(\widehat{c})(z^t,\eta^h) = \frac{\widehat{c}(z^t,\eta^h)^{1-\gamma}}{1-\gamma} + \sum_{z_{t+1}}\widehat{\beta}(z_{t+1},z^t)\widehat{\phi}(z_{t+1}|z^t)\sum_{\eta_{h+1}}\varphi(\eta_{h+1}|\eta^h)U(\widehat{c})(z^{t+1},\eta^{h+1}) \tag{C.1}$$

rank consumption plans identically as in the original economy.

When there is predictability in aggregate consumption growth, shocks to expected growth manifest themselves as time discount rate shocks in the stationary economy. If aggregate growth shocks are i.i.d. over time, then the stationary time discount factor is constant and given by:

$$\widehat{\beta} = \beta\sum_{z_{t+1}}\phi(z_{t+1})\lambda_{t+1}(z_{t+1})^{1-\gamma}. \tag{C.2}$$

This i.i.d. assumption on aggregate growth shocks is the assumption we will make, noting that it can easily be relaxed. In what follows, we also assume that aggregate factor shares are constant: $\alpha_t(z_t) = \alpha,\ \forall t$. By definition, labor income shares average to one across households:

$$\sum_{t_0\geq 1}\sum_{\eta^h}\varphi(\eta^h|\eta_0)\widehat{y}_t(\eta^h) = 1,\ \forall t.$$

## C.3 Equilibrium in the Stationary Economy

In the stationary economy, agents trade a single risk-free bond and a stock. Both securities have the same returns in the absence of aggregate risk. The stock yields a dividend $\alpha$ in each period. Given initial financial wealth $\theta_0$, interest rates $\widehat{R}_t$ and stock prices $\widehat{v}_t$, households choose consumption $\{\widehat{c}_t(\theta_0,\eta^h)\}$, bond positions $\{\widehat{a}_t(\theta_0,\eta^h)\}$, and stock positions $\{\widehat{\sigma}_t(\theta_0,\eta^h)\}$ to maximize expected utility (C.1) subject to the budget constraint:

$$\widehat{c}_t(\eta^h) + \frac{\widehat{a}_t(\theta_0,\eta^h)}{\widehat{R}_t} + \widehat{\sigma}_t(\theta_0,\eta^h)\widehat{v}_t = (1-\alpha)\widehat{y}_t(\eta^h) + \widehat{a}_{t-1}(\theta_0,\eta^{h-1}) + \widehat{\sigma}_{t-1}(\theta_0,\eta^{h-1})(\widehat{v}_t+\alpha),$$

and subject to borrowing constraints:

$$\frac{\widehat{a}_t(\theta_0,\eta^h)}{\widehat{R}_t} + \widehat{\sigma}_t(\theta_0,\eta^h)\widehat{v}_t \geq \widehat{K}_t(\eta^h), \quad \forall\eta^h$$

$$\widehat{a}_t(\theta_0, \eta^h) + \widehat{\sigma}_t(\theta_0, \eta^h)(\widehat{\nu}_{t+1} + \alpha) \geq \widehat{M}_t(\eta^h), \quad \forall \eta^h.$$

**Definition 2.** For a given initial distribution of wealth $\Theta_0$, a Bewley equilibrium is a list of consumption choices $\{\widehat{c}_t(\theta_0, \eta^h)\}$, bond positions $\{\widehat{a}_t(\theta_0, \eta^h)\}$, and stock positions $\{\widehat{\sigma}_t(\theta_0, \eta^h)\}$ as well as stock prices $\widehat{\nu}_t$, and interest rates $\widehat{R}_t$ such that each household maximizes its expected utility, and asset markets and goods markets clear.

$$\sum_{t_0 \geq 1} \int \sum_{\eta^h} \varphi(\eta^h | \eta_{t_0}) \widehat{a}_t(\theta_0, \eta^h) d\Theta_0 = 0,$$

$$\sum_{t_0 \geq 1} \int \sum_{\eta^h} \varphi(\eta^h | \eta_{t_0}) \widehat{\sigma}_t(\theta_0, \eta^h) d\Theta_0 = 1.$$

$$\sum_{t_0 \geq 1} \int \sum_{\eta^h} \varphi(\eta^h | \eta_{t_0}) \widehat{c}_t(\theta_0, \eta^h) d\Theta_0 = 1.$$

In the stationary economy, the return on the aggregate stock equals the risk-free rate:

$$\widehat{R}_t = \frac{\widehat{\nu}_{t+1} + \alpha}{\widehat{\nu}_t}. \tag{C.3}$$

The equilibrium stock price equals the present discounted value of the dividends:

$$\widehat{\nu}_t = \sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{t \to t+\tau} \alpha,$$

discounted at the cumulative gross risk-free rate, defined as: $\widehat{R}_{t \to t+T} = \Pi_{k=0}^{T} \widehat{R}_{t+k}$. Note that $\widehat{R}_{t \to t} = \widehat{R}_t$ and define $\widehat{R}_{t \to t-1} = 1$. Since both assets, the stock and the risk-free bond, earn the same risk-free rate of return in the stationary economy, households are indifferent between them. This indifference extends to any other assets with different durations since interest rates are deterministic in the stationary economy.

### C.3.1 Connection with the Equilibrium in the Growing Economy

We can map the equilibrium in the stationary economy into an equilibrium in the stochastically growing economy.

**Proposition 2.** If $\{\widehat{c}_t(\theta_0, \eta^h), \widehat{a}_t(\theta_0, \eta^h), \widehat{\sigma}_t(\theta_0, \eta^h)\}$ and $\{\widehat{\nu}_t, \widehat{R}_t\}$ are a Bewley equilibrium, then $\{c_t(\theta_0, z^t, \eta^h), a_t(\theta_0, z^t, \eta^h, z_{t+1}), \sigma_t(\theta_0, z^t, \eta^h)\}$ as well as asset prices $\{\nu_t(z^t), q_t(z^t, z_{t+1})\}$ are an equilibrium of the stochastically growing economy with:

$$\begin{aligned}
c_t(\theta_0, z^t, \eta^h) &= \widehat{c}_t(\theta_0, \eta^h) e_t(z^t) \\
a_t(\theta_0, z^t, \eta^h; z_{t+1}) &= \widehat{a}_t(\theta_0, \eta^h; z_{t+1}) e_t(z^t) \\
\sigma_t(\theta_0, z^t, \eta^h) &= \widehat{\sigma}_t(\theta_0, \eta^h) \\
\nu_t(z^t) &= \widehat{\nu}_t e_t(z^t) \\
q_t(z^t, z_{t+1}) &= \frac{\widehat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \frac{1}{\widehat{R}_t}.
\end{aligned}$$

The proof is provided in Krueger and Lustig (2010).

The last equation in the proposition above implies the following relationship between the interest rate in the growing economy ($R_t$) and the stationary economy ($\widehat{R}_t$):

$$R_t = \left( \sum_{z_{t+1}} q_t(z^t, z_{t+1}) \right)^{-1} = \left( \sum_{z_{t+1}} \frac{\widehat{\phi}(z_{t+1})}{\lambda(z_{t+1})} \right)^{-1} \widehat{R}_t. \tag{C.4}$$

or, plugging in for $\widehat{\phi}(z_{t+1}|z^t)$:

$$\widehat{R}_t = \frac{E_t\left[\lambda_{t+1}^{-\gamma}\right]}{E_t\left[\lambda_{t+1}^{1-\gamma}\right]} R_t$$

### C.3.2 Log-normal Growth

Consider a special case where the aggregate endowment growth rate $\lambda_t$ is i.i.d. log-normally distributed:

$$log(\lambda_t) \sim \mathcal{N}(g, \sigma_\lambda^2)$$

Then:

$$E_t[\lambda_{t+1}^{-\gamma}] = E_t[\exp(-\gamma \log(\lambda_{t+1}))] = \exp(-\gamma g + 0.5\gamma^2 \sigma_\lambda^2)$$

and

$$E_t[\lambda_{t+1}^{1-\gamma}] = E_t[\exp((1-\gamma) \log(\lambda_{t+1}))] = \exp((1-\gamma) g + 0.5(1-\gamma)^2 \sigma_\lambda^2)$$

We obtain

$$\widehat{R} = \frac{\exp(-\gamma g + 0.5\gamma^2 \sigma_\lambda^2)}{\exp((1-\gamma) g + 0.5(1-\gamma)^2 \sigma_\lambda^2)} R_t = \frac{R}{G},$$

where

$$G = \exp(g + 0.5\sigma_\lambda^2 - \gamma \sigma_\lambda^2)$$

which recovers equation (12) in the main text. (Recall that the main text refers to the interest rate in the growing economy as $R_g$ and to the interest rate in the stationary economy as $R$.)

Using lowercase letters to denote logs:

$$\hat{r} = r - g - 0.5\sigma_\lambda^2 + \gamma\sigma_\lambda^2$$

**Changes in Interest Rates** Now consider the relationship between the time-series change in the interest rate in the growing economy and the time-series change in the interest rate in the stationary economy. Denote the initial and new steady states by the subscripts 0 and $T$. Assume that the growth rate uncertainty does not change between steady states, but only the subjective time discount factor and/or the expected growth rate of the economy:

$$\hat{r}_T - \hat{r}_0 = (r_T - r_0) - (g_T - g_0)$$

The interest rate in the growing economy can be written, from the first-order condition, as:

$$r_t = -\log(\beta) + \gamma g - 0.5\gamma^2\sigma_\lambda^2.$$

Under the maintained assumption of no change in growth uncertainty, the change in interest rates in the growing economy is:

$$r_T - r_0 = -\log(\beta_T) + \log(\beta_0) + \gamma(g_T - g_0)$$

.

The change in rates in the stationary economy is lower by the change in the growth rate in the actual economy. We can also write this as:

$$\hat{r}_T - \hat{r}_0 = -(\log(\beta_T) - \log(\beta_0)) + (\gamma - 1)(g_T - g_0)$$

The change in the equilibrium interest rate in the stationary economy reflects either a change in the subjective time discount factor in the growing economy or a change in the expected growth rate of the economy or a combination of the two. The effect of a change in the expected growth rate on the interest rate depends on the inter-temporal elasticity of substitution (IES) $\gamma^{-1}$. If the IES is smaller than 1 ($\gamma > 1$), then a decrease in the expected growth rate results in a decrease in the interest rate; the income effect dominates the substitution effect.

Last, we can compute the impact on $\hat{\beta}$. Since

$$\hat{\beta} \quad = \quad \beta E_t[\lambda_{t+1}^{1-\gamma}] = \beta \exp\left\{(1-\gamma)g + 0.5(1-\gamma)^2\sigma_\lambda^2\right\}$$

In logs:

$$\log \hat{\beta} \quad = \quad \log \beta + (1-\gamma)g + \frac{1}{2}(1-\gamma)^2\sigma_\lambda^2$$

Under the maintained assumption that $\sigma_\lambda^2$ does not change between 0 and $T$, we have that:

$$\log \hat{\beta}_T - \log \hat{\beta}_0 = \log \beta_T - \log \beta_0 + (1-\gamma)(g_T - g_0)$$

implying that

$$\log(\hat{R}_T \hat{\beta}_T) - \log(\hat{R}_0 \hat{\beta}_0) = 0.$$

The change in $\log \hat{\beta}$ is of the same magnitude and opposite sign as the change in $\hat{r}$.

In the calibrated model, we envision the decline in interest rates in the data is driven by an increase in $\beta$ so that:

$$\hat{r}_T - \hat{r}_0 = r_T - r_0 = -(\log(\beta_T) - \log(\beta_0)) = -(\log \hat{\beta}_T - \log \hat{\beta}_0) = 4.48\%.$$

## C.4 Wealth Accounting

What is the right discount rate to use when measuring household wealth? If we want a wealth measure that can be aggregated, we have to use the same discount rate $\widehat{R}$ for all claims.

**Proposition 3.** At time 0, the financial wealth of each household equals the present discounted value of future consumption minus future labor income.

$$\theta_0 \quad = \quad \sum_{\tau=0}^{\infty} \sum_{\eta^\tau} \frac{\varphi(\eta^\tau)}{\widehat{R}_{0\to\tau-1}} \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)$$

The proposition follows directly from iterating forward on the one-period budget constraint. In this iteration, we take expectations over financial wealth in all future states using the objective probabilities of the idiosyncratic events $\varphi(\eta^\tau)$, and discount by the cumulative risk-free rate $\widehat{R}_{0\to\tau-1}$. Aggregate financial wealth in the economy in period 0 is given by:

$$\int \theta_0 d\Theta_0 = \int (\widehat{a}_{-1}(\theta_0) + \widehat{\sigma}_{-1}(\theta_0)\widehat{\nu}_0) d\Theta_0 = 0 + 1\widehat{\nu}_0,$$

where we have used market clearing in the bond and stock markets at time 0.

Aggregating the cost of the excess consumption plan across all households, using the fact that labor income shares average to 1, and imposing goods market clearing at time 0, we get:

$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \alpha = \widehat{v}_0.$$

The aggregate cost of households' excess consumption plan, or households' aggregate financial wealth, exactly equals the stock market value $\widehat{v}_0$, the only source of net financial wealth in the economy. This result relies on market clearing:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 = \alpha,$$

at each time $t$, because $\int \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 = 1$ from market clearing, and the labor income shares sum to one as well.

The choice of the actual probability measure $\varphi(\cdot)$ and rate $\widehat{R}$ to compute an individual's human capital, the expected present discounted value of her labor income stream, may seem arbitrary. After all, claims to labor income are not traded in this model and markets are incomplete. The key insight is that, using any other pricing kernel to discount individual labor income and consumption streams may result in a value of aggregate financial wealth different from the value of the Lucas tree. To see this, consider using a distorted measure $\psi(\eta^\tau)\varphi(\eta^\tau)$ different from the actual measure $\varphi(\eta^\tau)$, where the household-specific wedges satisfy $\mathbb{E}_0[\psi_t] = 1, \forall t$. Under this different measure, the goods markets do not clear and the labor shares do not sum to one, unless the household-specific wedges do not covary with consumption and income shares:

**Proposition 4.** Wealth measures aggregate if and only if the following orthogonality conditions holds for the househehold-specific wedges and household consumption and income:

$$Cov_0\left(\psi_t, \widehat{c}_t\right) = 0, \quad Cov_0\left(\psi_t, \widehat{y}_t\right) = 0.$$

For all other wedge processes $\psi_t(\eta^\tau)$, the resource constraint is violated:

$$\int \sum_{\eta^\tau} \psi(\eta^\tau)\varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right) d\Theta_0 \neq \alpha,$$

It is common in the literature to use the household's own IMRS to compute human capital. The household's IMRS is a natural choice because it ties the valuation of human wealth directly to welfare. However, this approach does not lend itself to aggregation. The wedges

$$\psi(\eta^{t+1}) = \frac{u'(\widehat{c}(\eta_{t+1}, \eta^t))}{u'(\widehat{c}_t(\eta_0))},$$

do not satisfy the zero covariance restrictions of the proposition. Imperfect consumption insurance implies that:

$$Cov_0(\psi_t, \widehat{c}_t) \leq 0, \quad Cov_0(\psi_t, \widehat{y}_t) \leq 0.$$

**Proposition 5.** If the cross-sectional covariance between the household-specific wedges and consumption is negative ($Cov_0(\psi_t, \widehat{c}_t) \leq 0$), then the aggregate valuation of individual wealth is less than the market's valuation of total wealth.

When aggregating, this pricing functional undervalues human wealth and therefore also total wealth.[13] In sum, while pricing claims to consumption and labor income using the household's IMRS is sensible from a welfare perspective, this approach does not lend itself to wealth accounting and aggregation.

### C.5 Interest Rate Decline

We now analyze the main exercise of the paper, which is to let the economy undergo an unexpected and permanent decrease in the interest rate ("MIT shock"). We study the implications for inequality in financial wealth.

Since interest rates are endogenously determined, we generate the decline in the equilibrium real rate in the stationary model, $\widehat{R}$, through increase in the deflated subjective time discount factor, $\widehat{\beta}$. As discussed in Section C.3.2, the latter arises either from an increase in the subjective time discount factor in the economy with growth, $\beta$, a decline in the expected rate of growth of the aggregate endowment, $E[\lambda]$ (or equivalently $G$), or some combination of the two. We focus on the case of an increase in the subjective time discount factor, but the theoretical results go through if all or some of the change in interest rates comes from a decline in expected growth. We denote the equilibrium of the stationary economy under high interest rates with a hat ($\widehat{x}$) and the equilibrium of the stationary economy under low interest rates with a tilde ($\widetilde{x}$).

It is natural to ask whether the equilibrium consumption allocation $\{\widehat{c}_t(\theta_0, \eta^t)\}$ that prevailed in the economy with high rates is still an equilibrium after the change in interest rates. Given that the time discount factor of all agents increased by the same amount, there should be no motive to trade away from these allocations: $\widetilde{\beta}\widetilde{R} = \widehat{\beta}\widehat{R} = 1$. The following proposition shows that the old consumption allocation is indeed still an equilibrium in the low interest rate economy, provided that initial financial wealth is scaled up for every household.

**Proposition 6.** If the allocations and asset market positions $\{\widehat{c}_t(\theta_0, \eta^t), \widehat{a}_t(\theta_0, \eta^t), \widehat{\sigma}_t(\theta_0, \eta^t)\}$ and asset prices $\{\widehat{v}_t, \widehat{R}_t\}$ are a Bewley equilibrium in the economy with $\widehat{\beta}$ and natural borrowing limits

[13] Since the factor shares are constant, the consumption claim is in the span of traded assets. Financial wealth is the value of the Lucas tree, which equals $\alpha$ times the value of a claim to total consumption.

$\{\widehat{K}_t(\eta^t)\}$,

$$\widehat{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \widehat{R}^{-1}_{t\to\tau-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t)(1-\alpha)\widehat{y}_\tau(\eta^\tau),$$

then the allocations and asset market positions $\{\widehat{c}_t(\widetilde{\theta}_0, \eta^t), \widehat{a}_t(\widetilde{\theta}_0, \eta^t), \widehat{\sigma}_t(\widetilde{\theta}_0, \eta^t)\}$ and asset prices $\{\widetilde{v}_t, \widetilde{R}_t\}$ will be an equilibrium of the economy with $\widetilde{\beta}$ and natural borrowing limits $\{\widetilde{K}_t(\eta^t)\}$,

$$\widetilde{K}_t(\eta^t) = \sum_{\tau=t}^{\infty} \widetilde{R}^{-1}_{t\to\tau-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t)(1-\alpha)\widehat{y}_\tau(\eta^\tau),$$

asset prices are given by

$$\widetilde{\beta}\widetilde{R}_t \quad = \quad \widehat{\beta}\widehat{R}_t, \text{ and } \widetilde{v}_t = \sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{t\to t+\tau}\alpha,$$

and every household's initial wealth is adjusted as follows:

$$\widetilde{\theta}_0 = \theta_0 \frac{\sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{0\to\tau} \sum_{\eta^\tau} \varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)}{\sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{0\to\tau} \sum_{\eta^\tau} \varphi(\eta^\tau)\left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)}.$$

The proof is below.

Aggregate financial wealth undergoes an adjustment equal to the ratio of the price of two perpetuities:

$$\frac{\sum_{\tau=0}^{\infty} \widetilde{R}^{-1}_{0\to\tau}}{\sum_{\tau=0}^{\infty} \widehat{R}^{-1}_{0\to\tau}} = \frac{\widetilde{v}_0}{\widehat{v}_0}.$$

Intuitively, with lower interest rates, all asset prices are higher than in the high-rate economy. The Lucas tree becomes more valuable. A fraction $1-\alpha$ of this tree reflects aggregate human wealth, the remaining fraction is aggregate financial wealth.

Each individual's financial wealth adjustment differs, and depends on the expected discounted value of the same future excess consumption plan discounted at different rates. The higher one's expected future excess consumption, the larger the initial financial wealth adjustment needed to implement the old equilibrium allocation.

**Characterizing Interest Rate Sensitivity Using Duration of Excess Consumption** To a first-order approximation, i.e., for a small change in the interest rate, the adjustment in initial financial wealth needed for agents to keep their initial consumption plan is given by the duration of their planned consumption in excess of labor income. This is the duration households will need in their net financial assets in order to be fully hedged against interest rate risk.

Define the duration of a household's excess consumption plan at time 0, following the realization of the idiosyncratic labor income shock $\eta_0$, as follows:

$$D^{c-y}(\theta_0, \eta_0) = \frac{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau|\eta_0} \tau \widehat{R}_{0\to\tau}^{-1} \varphi(\eta^t|\eta_0) \left(\widehat{c}_\tau(\eta^\tau|\eta_0) - (1-\alpha)\widehat{y}(\eta^\tau|\eta_0)\right)}{\sum_{\tau=0}^{\infty} \sum_{\eta^\tau|\eta_0} \varphi(\eta^t|\eta_0) \widehat{R}_{0\to\tau}^{-1} \left(\widehat{c}_\tau(\eta^\tau|\eta_0) - (1-\alpha)\widehat{y}(\eta^\tau|\eta_0)\right)}$$

The duration measures the sensitivity of the cost of its excess consumption plan to a change in the interest rate. In our endowment economy, aggregate consumption is fixed. We are interested in the valuation effects of interest rate changes.

The duration of the excess consumption claim equals the value-weighted difference of the duration of the consumption claim and that of the labor income claim:

$$D^{c-y} = \frac{P_0^c}{P_0^{c-y}} D^c - \frac{P_0^y}{P_0^{c-y}} D^y.$$

where $P_0^{c-y} = \theta_0$ is household financial wealth, $P_0^y$ is human wealth, and $P_0^c$ is total household wealth, the sum of financial and human wealth. Households with a high positive duration of excess consumption face a large increase in the cost of their consumption plan when interest rates go down, insofar that this increased cost is not offset fully by the increase in their human wealth.

The duration of the aggregate excess consumption claim, the aggregate duration for short, equals:

$$D^a = \frac{\sum_{\tau=0}^{\infty} \tau \widehat{R}_{0\to\tau}^{-1}}{\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1}}$$

This is the duration of a claim to aggregate consumption minus aggregate labor income, or equivalently to aggregate financial income. It is the duration of a perpetuity in the stationary economy. Recall that $\widehat{\nu}_0 = \nu_0 = \alpha \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau}^{-1}$ denotes aggregate financial wealth.

**Proposition 7.** The aggregate duration equals the wealth-weighted average duration of households' excess consumption claims:

$$D^a = \int D^{c-y}(\theta_0, \eta_0) \frac{\theta_0}{\nu_0} d\Theta_0.$$

The proof follows directly from the definition of the household specific duration measure and market clearing.

The next proposition is the main result. It shows that, when households that are richer than average tend to have excess consumption plans of higher duration, then the (equally-weighted) average household's excess consumption plan duration is smaller than the aggregate duration.

**Proposition 8.** If $cov(\theta_0, D^{c-y}(\theta_0)) > 0$ then $\int D^{c-y}(\theta_0, \eta_0) d\Theta_0 \leq D^a$ and lower interest rates increase financial wealth inequality when households are fully hedged.

The proof follows from recognizing the following relationship between (cross-sectional) expectations and covariances:

$$D^a = \mathbb{E}\left[\frac{\theta_0}{\nu_0^a} D^{c-y}(\theta_0,\eta_0)\right] = \mathbb{E}\left[D^{c-y}(\theta_0,\eta_0)\right] + cov\left[\frac{\theta_0}{\nu_0}, D^{c-y}(\theta_0,\eta_0)\right].$$

The proposition says that under the covariance condition, if all households are perfectly hedged in their portfolio, then wealth inequality should increase when rates decline.

**Ex-Ante Identical Households** In this class of Bewley models, if agents are ex-ante identical, agents with low financial wealth have encountered a bad history of labor income shocks. If labor income is highly persistent, their labor income is low today and in the near future relative to labor income in the distant future (because of mean-reversion). This pattern makes the duration of their labor income stream high. But since the household is smoothing consumption inter-temporally, $D^c < D^y$. As a result, low-wealth agents tend to have low duration of their excess consumption plan. Conversely, rich agents have high labor income and high excess consumption duration. Consumption smoothing is the force that makes the assumption of a positive covariance between the level of financial wealth and the duration of excess consumption satisfied in a Bewley model where the only source of heterogeneity is income shock realizations. It follows immediately from Proposition 8 that the decline in rates (i) increases the cost of the excess consumption plan for the aggregate (per capita) value-weighted household by more than the cost for the equally-weighted average household, and (ii) increases financial wealth inequality. Put differently, in a model where all households are exactly equally well off after the change in rates by construction, i.e., they are perfectly hedged, financial wealth inequality should increase when rates go down.

Low-financial wealth households in a Bewley model have high-duration human wealth, which provides a natural interest rate hedge. High financial-wealth households have low-duration human wealth and need to increase financial wealth by more when rates decline to be able to afford the old consumption plan.

**Ex-Ante Heterogeneous Households** The insights of this normative proposition apply more broadly to a richer model with ex-ante heterogeneity across households, for example because agents go through a life cycle and differ by age.

**Proposition 9.** If $cov(\theta_t, D_t^{c-y}(\theta_{t_0})) > 0$ then the average duration is lower than the aggregate duration, $\sum_{t_0} \int D_t^{c-y}(\theta_0,\eta_0) d\Theta_{t_0} \leq D_t^a$ and lower interest rates increase financial wealth inequality when households are fully hedged.

We check this condition in the calibrated version of the model.

Real-world households may not be fully hedged, unlike the households in the Bewley model. The actual duration of the household's financial assets in the data, denoted $D^\theta$, can differ from the duration of the excess consumption claim $D^{c-y}$ in the model where households are fully hedged. Section 2 of the paper considers a calibrated life-cycle version of the Bewley model with overlapping generations to assess how well households are hedged against interest rate risk.

## C.6 Proofs of Propositions in this Appendix

### C.6.1 Proof of proposition 3

*Proof.* The one-period budget constraint:

$$\widehat{c}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t)}{\widehat{R}_t} + \widehat{\sigma}_t(\eta^t)\widehat{v}_t = (1-\alpha)\widehat{y}_t(\eta^t) + \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha),$$

can be restated, using equation (C.3), as:

$$\widehat{c}_t(\eta^t) - (1-\alpha)\widehat{y}_t(\eta^t) + \frac{\widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1}+\alpha)}{\widehat{R}_t} = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha). \tag{C.5}$$

Rewriting (C.5) one period later:

$$\widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2}+\alpha)}{\widehat{R}_{t+1}} = \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1} + \alpha).$$

Multiply this equation by $\varphi(\eta_{t+1}|\eta^t)$ and sum across all states $\eta_{t+1}$ to obtain:

$$\begin{aligned} & \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) + \frac{\widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2}+\alpha)}{\widehat{R}_{t+1}} \right) \\ = \ & \widehat{a}_t(\eta^t) + \widehat{\sigma}_t(\eta^t)(\widehat{v}_{t+1}+\alpha), \end{aligned}$$

where we used the fact that $\sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) = 1$ on the right-hand side. Next, substitute this expression back into (C.5) to obtain:

$$\begin{aligned} & \widehat{c}_t(\eta^t) - (1-\alpha)\widehat{y}_t(\eta^t) + \widehat{R}_t^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{c}_{t+1}(\eta^{t+1}) - (1-\alpha)\widehat{y}_{t+1}(\eta^{t+1}) \right) \\ & + \widehat{R}_{t\to t+1}^{-1} \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \left( \widehat{a}_{t+1}(\eta^{t+1}) + \widehat{\sigma}_t(\eta^{t+1})(\widehat{v}_{t+2}+\alpha) \right) = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t+\alpha). \end{aligned}$$

Define financial wealth, scaled by the aggregate endowment, as:

$$\widehat{\theta}_t = \widehat{a}_{t-1}(\eta^{t-1}) + \widehat{\sigma}_{t-1}(\eta^{t-1})(\widehat{v}_t + \alpha).$$

Continuing the forward substitution, we end up with the following expression:

$$\widehat{\theta}_t = \sum_{\tau=t}^{\infty} \widehat{R}_{t\to\tau-1}^{-1} \sum_{\eta^\tau|\eta^t} \varphi(\eta^\tau|\eta^t) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right).$$

where $\varphi(\eta^t|\eta^t) = 1$. Financial wealth must equal the cost of the household's excess consumption plan, where excess refers to the part not paid for with labor income. Noting that $e_0 = 1$ so that $\widehat{\theta}_0 = \theta_0$, writing this expression at time zero:

$$\theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right)$$

recovers the statement of the proposition. □

### C.6.2 Proof of Proposition 4

*Proof.* We note that the cross-sectional expectation of the product can be decomposed in the standard way:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau)\right) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \mathbb{C}ov_0[\psi_\tau, c_\tau] + \mathbb{E}_0\left[\psi_\tau\right]\mathbb{E}_0\left[c_\tau\right].$$

If the orthogonality condition is satisfied, then the following result obtains:

$$\int \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau) \left(\widehat{c}_\tau(\eta^\tau)\right) d\Theta_0 = \mathbb{E}_0[\psi_\tau c_\tau] = \mathbb{E}_0[\psi_\tau]\mathbb{E}_0[c_\tau] = \mathbb{E}_0[c_\tau] = 1,$$

because $\mathbb{E}_0[\psi_t] = 1$. □

### C.6.3 Proof of Proposition 5

*Proof.* This inequality $0 \geq Cov(\psi_t, \widehat{c}_t)$ directly implies that the following inequalities obtain:

$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 \leq \int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{c}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1},$$
$$\int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\psi(\eta^\tau)\widehat{y}_\tau(\eta^\tau) d\Theta_0 \leq \int \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^\tau)\widehat{y}_\tau(\eta^\tau) d\Theta_0 = \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}.$$

As a result, this new measure implies an aggregate value of individual wealth that falls short of total wealth, $\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$. Note that even though this claim to total consumption is itself not traded, the Lucas tree is a claim to $\alpha$ of the same cash flow stream. The market value of the Lucas tree is $\alpha \sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$, and hence the value of total wealth has to be $\sum_{\tau=0}^{\infty} \widehat{R}_{0\to\tau-1}^{-1}$. □

### C.6.4 Proof of proposition 6

*Proof.* An unconstrained household's Euler equation in the high-growth economy is given by:

$$1 = \widehat{\beta}\widehat{R}_t \sum_{\eta_{t+1}} \varphi(\eta_{t+1}|\eta^t) \frac{u'(\widehat{c}(\eta_{t+1}, \eta^t))}{u'(\widehat{c}_t(\eta^t))}.$$

This Euler equation is satisfied because the allocations and prices constitute a Bewley equilibrium in the high-growth economy. This household's Euler equation in the new economy with lower interest rates is still satisfied at the old consumption allocation. This can be seen by plugging in the new equilibrium interest rates:

$$\widetilde{R}_t\widetilde{\beta} = \widehat{\beta}\widehat{R}_t,$$

to recover the unconstrained household's Euler equation in the low-growth economy:

$$1 = \widetilde{\beta}\widetilde{R}_t \sum_{\eta_{t+1}} \phi(\eta_{t+1}|\eta_t) \frac{u'(\widehat{c}(\eta^t, \eta_{t+1})}{u'(\widehat{c}_t(\eta^t))}.$$

We allocate the following amount of financial wealth at time 0 to ensure the household can afford the same consumption plan:

$$\widetilde{\theta}_0(\theta_0, \eta_0) \quad = \quad \sum_{\tau=0}^{\infty} \widetilde{R}_{0\to\tau-1}^{-1} \sum_{\eta^\tau} \varphi(\eta^t) \left(\widehat{c}_\tau(\eta^\tau) - (1-\alpha)\widehat{y}_\tau(\eta^\tau)\right).$$

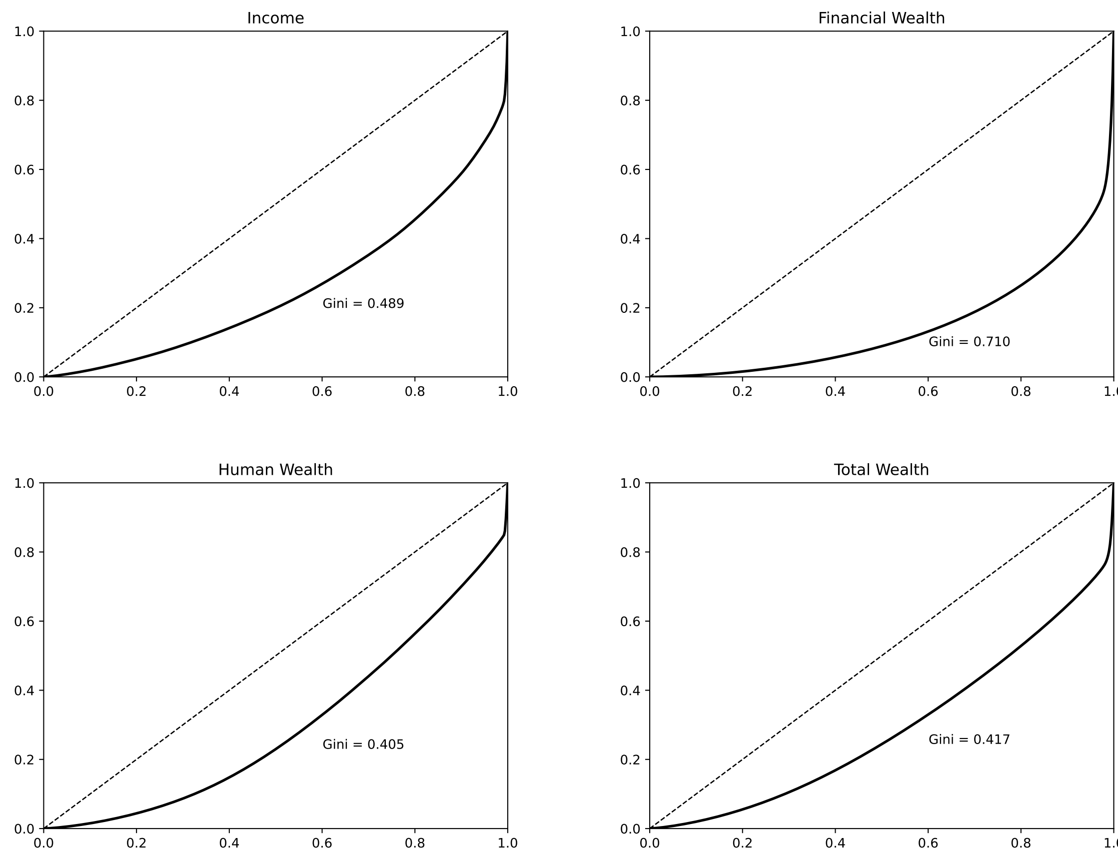
Aggregating this initial financial wealth across households:

$$\int \widetilde{\theta}_0 d\Theta_0 \quad = \quad \alpha \sum_{\tau=0}^{\infty} \widetilde{R}_{0\to\tau}^{-1} = \tilde{v}_0,$$

where we have used the goods market clearing condition and the definition of labor income shares. The last equation shows that the new allocation of initial financial wealth uses up all aggregate financial wealth in the economy. Finally, note that the natural borrowing constraints are not binding in the high-growth economy. They remain non-binding in the low-growth economy because consumption is nonnegative. Hence, the allocations are feasible, and they satisfy the sufficient conditions for optimality. □

# D Additional Model Results

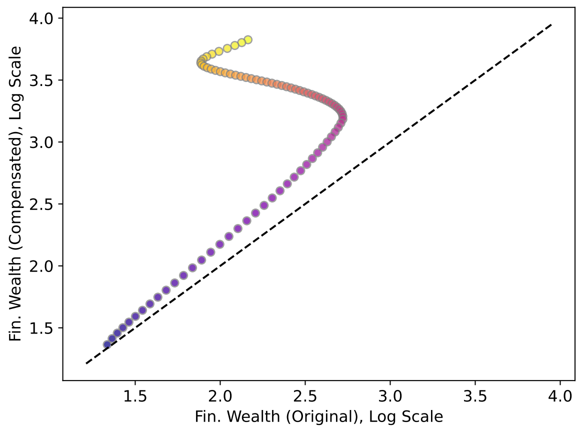
Figure D1: Lorenz Curves



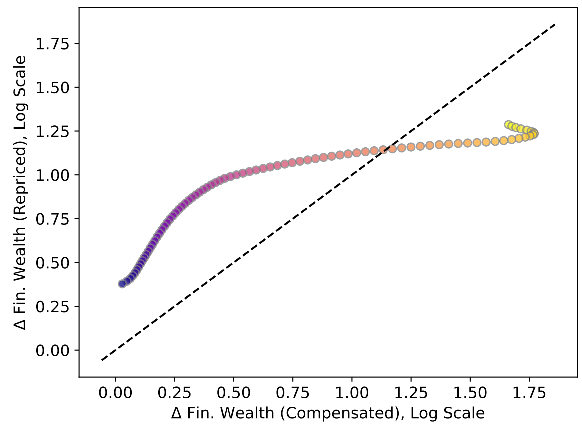

*Note:* This figure plots the Lorenz curve for each variable, obtained from a long simulation of the model.

## Figure D2: Scatterplots by Age: Medians

(a) Compensated vs. Original



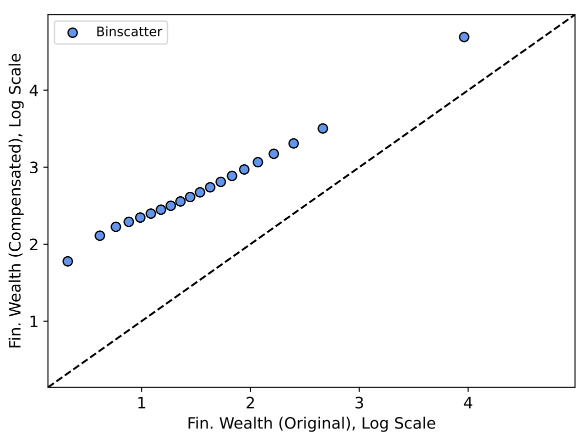

(b) Compensated vs. Repriced



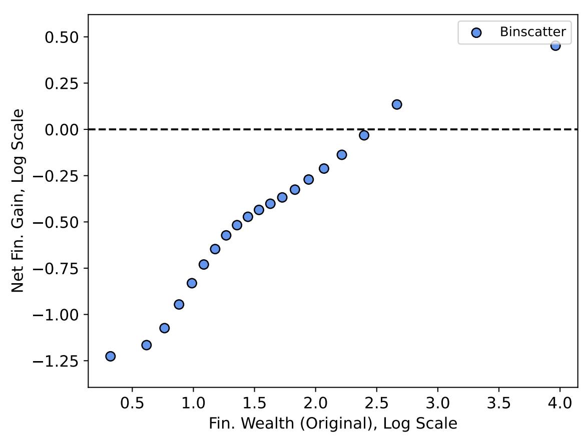

*Note:* Panel (a) plots the distribution of original financial wealth against the distribution of compensated financial wealth by age. Each dot represents the median value for one year of age, with the lightest (yellow) dots representing the youngest agents and the darkest (purple) dots representing the oldest agents. Both variables are plotted using the transform $\log(1 + x)$. Panel (b) similarly plots medians for one-year bins of the change in financial wealth under repricing, compared to the change in financial wealth under the compensated distribution, both using the transform $\log(1 + x)$ before differencing. The dashed line represents equality between the x and y axes.

## Figure D3: Binscatters by Wealth, Controlling for Age

(a) Compensated vs. Original




(b) Compensated vs. Repriced




*Note:* Panel (a) plots the distribution of original financial wealth against the distribution of compensated financial wealth by age. Each dot represents 5% of the original financial wealth distribution. Both variables are plotted using the transform $\log(1 + x)$. Panel (b) similarly plots medians for 5% bins of the change in financial wealth under repricing, compared to the change in financial wealth under the compensated distribution, both using the transform $\log(1 + x)$ before differencing. The dashed line represents equality between the x and y axes.